Determinacy without the Taylor Principle

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Outline

A Perennial Issue: Indeterminacy in a Monetary Economy

- Core questions of the monetary economics depends on equilibrium selection:
  - What determines the price level?
  - Can monetary policy regulate AD and inflation?
  - Does the ZLB trigger a deflationary spiral?

- Basic problem (back to Sargent & Wallace, 75):
  - **Same path for $R \Rightarrow$ multiple bounded equilibrium paths for π and y**
  - **Different selections $\Rightarrow$ different answers to core monetary questions**

- State of the art: two alternatives
  - **Taylor principle** (TP, raise i more than 1-1 with π)
  - **Fiscal Theory of the Price Level** (FTPL, non-Ricardian fiscal policy)
  - Both boil down to **off-eq assumptions**, hard to test

A New Perspective

- Indeterminacy requires strong **intertemporal coordination ("infinite chain")**
  - Current agents respond to sunspots if future agents respond in a specific way.
  - Future agents respond only if they expect agents further in the future respond; and so on.
- **Small perturbations** in memory/coordination $\Rightarrow$ breaks the chain $\Rightarrow$ **determinacy**
  - Always selects the standard eq. (**MSV**), even with interest rate pegs
- A new perspective on **both** the **Taylor principle** and the **FTPL**
  - Recast Taylor principle as stabilization instead eq. selection
  - Reformulate FTPL outside the eq. selection logic

Outline

A Simplified NK Model: the AD Side

- Overlapping generations of consumers, each living two periods:

$$u(C^1_{i,t}) + \beta u(C^2_{i,t+1}) e^{-\rho_t},$$

- Budget

$$C^1_{i,t} + B_{i,t} = Y_t \quad \text{and} \quad C^2_{i,t+1} = Y_{t+1} + \frac{I_t}{\Pi_{t+1}} B_{i,t}$$

  - Young and old earn the same income Y_t
  - Young: strategic, optimally choose consumption given her expectations of Y and Π
    - borrow or save using a one-period nominal bond with 0 supply
  - Old: "robots," consumption adjusts to meet the budget

A Simplified NK Model: the IS Curve

- Log-linearized optimal consumption for the young

$$c^1_{i,t} = E_{i,t}\left[\frac{1}{1+\beta} y_t + \frac{\beta}{1+\beta} y_{t+1} - \frac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \rho_t)\right]$$

- Market clearing

$$y_t = \int c^1_{i,t} di = \int c^2_{i,t} di = c_t$$

- AD (where $\bar{E}_t[\cdot] = \int E_{i,t}[\cdot]\, di$ is the average expectations of the young)

$$c_t = \bar{E}_t\left[\frac{1}{1+\beta} c_t + \frac{\beta}{1+\beta} c_{t+1} - \frac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \rho_t)\right]$$

- AD under FIRE (Full-information rational expectations)

$$c_t = \mathbb{E}_t[c_{t+1}] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho_t)$$

- More generally: an example of an "intertemporal Keynesian cross"

A Simplified NK Model

1. DIS, with overlapping generations of consumers (who live for 2-periods):

$$c_t = \bar{E}_t \left[\frac{1}{1+\beta} y_t + \frac{\beta}{1+\beta} y_{t+1} - \frac{\beta}{1+\beta} \sigma (i_t - \pi_{t+1} - \rho_t) \right] \tag{DIS}$$

$$y_t = c_t$$

2. Phillips curve:

$$\pi_t = \kappa y_t + \xi_t \tag{PC}$$

3. Taylor rule ($\phi \geq 0$):

$$i_t = \phi \pi_t + z_t \tag{MP}$$

An Equivalent Representation

- Substituting monetary policy and Phillips curve in IS curve $\Rightarrow$

$$c_t = \bar{E}_t \left[(1-\delta_0)\theta_t + \delta_0 c_t + \delta_1 c_{t+1} \right]$$

  where $\{\theta_t\}$ is a transformation of $\{\rho_t, \xi_t, z_t\}$ and $\delta_0 \equiv \frac{1-\beta\sigma\phi\kappa}{1+\beta} < 1$ and $\delta_1 \equiv \frac{\beta+\beta\sigma\kappa}{1+\beta} > 0$

- NK economy = a game among consumers
  - δ_0 and δ_1 measure strategic complementarity within and across time
  - summarize all GE feedbacks: income$\leftrightarrow$spending, output$\leftrightarrow$inflation, MP response

FIRE Benchmark

- FIRE benchmark

$$\boxed{c_t = \theta_t + \delta \mathbb{E}_t[c_{t+1}]} \qquad \delta \equiv \frac{\delta_1}{1-\delta_0} = \frac{1+\kappa\sigma}{1+\phi\kappa\sigma} > 0$$

- Standard approach: **Taylor principle, unique bounded eq.** when

$$|\delta| < 1 \iff \phi > 1$$

- This paper: **unique bounded eq.** for any ϕ

Fundamentals, Sunspots, and the Equilibrium Concept

- Fundamentals & sunspots:

$$\theta_t \sim_{\text{i.i.d}} \mathcal{N}(0,1) \quad \text{and} \quad \eta_t \sim_{\text{i.i.d}} \mathcal{N}(0,1)$$

  - In paper: general stochasticity

- State of nature, or (infinite) history, at t:

$$h^t = \{\theta_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$$

- Equilibrium concept: **REE (based on potentially limited information about h^t)**

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k \theta_{t-k}$$

- Focus on bounded eq. (can be justified by escape clauses)

Outline

1 Introduction
2 A Simplified New Keynesian Model
3 The Standard Paradigm
4 Uniqueness with Fading Memory
5 The Full NK Model
6 Observation of Past Outcomes
7 Applied Lessons
8 Take-home Messages and Connections to the Literature

The Standard Paradigm

- **FIRE/perfect social memory benchmark:**

$$c_t = \theta_t + \delta(\phi)\mathbb{E}_t[c_{t+1}]$$

  - $\mathbb{E}_t[\cdot]$ is rational expectation conditional on entire history h^t

- **The MSV (minimum state variable) solution:**

$$c_t = c_t^F \equiv \theta_t$$

- **Is MSV the only solution?**
  - Standard: depends on the Taylor principle
  - Our perturbation: always

The Standard Paradigm

Proposition. Perfect Recall Benchmark

- When $\phi > 1$ (i.e., $\delta < 1$, Taylor principle), the MSV equilibrium is the unique eq
- When $\phi < 1$ (i.e., $\delta > 1$), there exist **a continuum of equilibria**

$$c_t = (1-b)c_t^F + bc_t^B + ac_t^\eta,$$

where $a, b \in \mathbb{R}$ are arbitrary scalars and

$$\underbrace{c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}}_{\text{sunspot eq.}} \quad \text{and} \quad \underbrace{c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k}}_{\text{backward-looking, pseudo-fundamental eq.}}$$

Understanding the Multiplicity

- Equilibrium condition (back-shifted)

$$c_{t-1} = \theta_{t-1} + \delta \mathbb{E}_{t-1}[c_t]$$

- Solving backwards (when $\phi < 1$, i.e., $\delta > 1$) :

$$\begin{aligned} \mathbb{E}_{t-1}[c_t] &= \delta^{-1}(c_{t-1} - \theta_{t-1}) \\ c_t &= \delta^{-1}(c_{t-1} - \theta_{t-1}) + \eta_t \\ c_t &= \underbrace{-\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k}}_{\substack{\text{backward-looking} \\ \text{pseudo-fundamental}}} + \underbrace{\sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k}}_{\text{sunspot}} \end{aligned}$$

- **Infinite chain of perfect intertemporal coordination:**
  - Current agents respond **to payoff-irrelevant variables** because they **expect to be rewarded by future agents**
  - Future agents themselves respond based on a similar expectation
  - ...

Outline

The First Perturbation: Fading Social Memory

- At every t, the young consumer learns (θ_t, η_t)
- With prob. λ, she learns nothing more
- With prob. $1-\lambda$, she inherits the info of a random old consumer

Assumption. Fading Social Memory

In every t, information set are given by

$$I_{i,t} = \{(\theta_t, \eta_t), \cdots, (\theta_{t-s}, \eta_{t-s})\},$$

where $s \in \{0, 1, \cdots\}$ is drawn from a geometric distribution with $\lambda \in (0,1)$.

- $s = 0$ with prob λ
- $s = 1$ with prob $(1-\lambda)\lambda$
- $s = k$ with prob $(1-\lambda)^k\lambda$

Determinacy without the Taylor Principle

- As $\lambda \to 0$, **almost all agents have *arbitrarily long* memory**
  - nearly perfect informed about $\{\theta_{t-k}, \eta_{t-k}, c_{t-k}, \pi_{t-k}\}$
- But for any $\lambda > 0$, zero mass of agents has *infinite* memory
  - $\lim_{k\to+\infty} \mu_k = 0$ where $\mu_k \equiv$ mass of agents that knows histories of length k or higher

Proposition. Determinacy without the Taylor Principle

With fading social memory, the **MSV solution** is the **unique equilibrium**

- **Regardless** of δ, or **equivalently MP** ϕ (e.g., even with pegs).
- No matter how slow the memory decay is (how small $\lambda > 0$ is).

Proof Sketch

- Simplification (here):
  - Abstract from fundamentals, let only shock be η_0 (effectively, focus on IRF of c_t to η_0)
  - Let $\delta_0 = 0$ and $\delta_1 = \delta$ and focus coordination cross time

- Maps to "twin" economy with infinite memory ($\lambda = 0$) but modified best response:

$$c_t = \delta \bar{E}_t [c_{t+1}] \implies c_t = \mu_t \delta \mathbb{E}[c_{t+1}]$$

  - $\lim_{t\to\infty} \mu_t = 0 \Rightarrow \mu_t \delta < 1$ for τ large enough
  - equivalently, RA economy with modified eigenvalue < 1
  - **perceived strategic complementarity eventually fades $\implies$ determinacy**

Logic

- I can see the current sunspot very clearly
- It would make sense to react if all future agents will keep responding to it **in perpetuity**
- But I worry that agents **far in the future will fail to do so**
  - either because they will forget it
  - or because they may worry that agents further into the future will forget it
- It therefore makes sense to ignore the sunspot

Outline

The Full NK Model: Three Equations

- Intertemporal Keynesian cross (proper DIS):

$$c_t = \mathscr{C}\left(\left\{\bar{E}_t[y_{t+k}]\right\}_{k=0}^{\infty}, \left\{\bar{E}_t[i_{t+k} - \pi_{t+k+1}]\right\}_{k=0}^{\infty}\right) + \rho_t$$

  - robust to incomplete info, summarizes consumer optimality & market clearing

- Standard forward-looking NKPC:

$$\pi_t = \kappa c_t + \beta E_t\left[\pi_{t+1}\right] + \xi_t$$

- Monetary policy ($\phi_c, \phi_\pi \geq 0$):

$$i_t = \phi_c c_t + \phi_\pi \pi_t + z_t$$

The Generalized Model and Nesting

- The generalized model:

$$c_t = \theta_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right]$$

  - our result only requires that the sum $\sum_{k=0}^{\infty} |\delta_k| < \infty$ & $\delta_0 < 1$

- Nests the previous micro-founded NK model with

$$\delta_k = (1 - \beta - \beta\sigma\phi_c)\beta^k - \sigma\kappa\phi_\pi\beta^{k+1}.$$

The Generalized Results mu

Proposition. Fading Memory Rules out Sunspot Volatility

With fading social memory ($\lambda > 0$), the equilibrium is unique and is given by the MSV solution.

Proof sketch: basically the same, focus on IRF of c_t to η_0

- "Twin" economy with attenuated strategic complementarity

$$c_t = \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right] \quad \Longrightarrow \quad c_t = \mu_t E_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right],$$

- Effective complementary, $\mu_t \left(\sum_{k=0}^{\infty} |\delta_k| \right)$, eventually < 1
- Unique eq. pinned down by iterating of best responses

Outline

Observation of Past Outcomes

- Baseline:
  - Preclude *direct* knowledge of past outcomes, such as c_{t-1}
  - Could **long memory of sunspots and past fundamentals** be efficiently "stored" in **short, public memory of past outcomes**?

- Sunspot eq. can be represented in **recursive form** as

$$c_t = \eta_t + \delta^{-1} c_{t-1}.$$

  - Supported by $I_{i,t} = \{\eta_t, c_{t-1}\}$
  - c_{t-1} serves as memory/coordination device

- Still takes a strong, **fragile**, form of **intertemporal coordination**

Proposition

Sunspot eq. **unravel** with tiny idiosyncratic noise in the observation of c_{t-1}

$$I_{i,t} = \{\eta_t, s_{i,t}\}, \qquad \text{with} \qquad s_{i,t} = c_{t-1} + \varepsilon_{i,t}.$$

Perfect Observation of Past Outcomes

- Chain of coordination can be broken even with **perfect knowledge of past** $\{c_{t-k}\}_{k=1}^{K}$,
- Add i.i.d. fundamental perturbation $\zeta_t \in [-\varepsilon, \varepsilon]$ (arbitrarily small) **known only to** t

$$c_t = \zeta_t + \delta \mathbb{E}_t [c_{t+1}]$$

- To support a sunspot eq, requires **perfect knowledge of** ζ_{t-1} **at** t

$$c_t = \eta_t + \delta^{-1} (c_{t-1} - \zeta_{t-1})$$

- But if ζ_{t-1} unknown to agents at t, the sunspot equilibrium collapses

Proposition

Sunspot eq. **unravel** with **bounded memory of what drives** $\{c_{t-k}\}_{k=1}^{K}$

$$I_t = \{\zeta_t\} \cup \{\eta_t ..., \eta_{t-K}\} \cup \{c_{t-1}, \cdots, c_{t-K}\}$$

Outline

A Smooth Taylor Principle

- **Recast Taylor principle as stabilization instead eq. selection**
- Our result removes the need for equilibrium selection
  but **leaves ample room for sunspot-like fluctuations** along the unique eq., e.g.,
  - overreaction to noisy public news (Morris-Shin, 02)
  - shocks to higher-order beliefs (Angeletos-La'O, 13)
  - bounded rationality (Angeletos & Sastry, 21)
- The slope of the Taylor rule admits a new function:
  - **regulates the magnitude of sunspot-like fluctuations** along the unique eq.
  - by regulating the overall complementarity in the economy

Fiscal Theory of Price Level

Proposition.

Assume first-order knowledge of government budget & market clearing + no rational confusion.

Then, gov debt and deficits are **payoff irrelevant**

- Regardless of memory, regardless of monetary/fiscal policy

- **Corollary:** eq. selected by FTPL is an eq. where **public debt serves as sunspots** and is not robust to our perturbations
- Fiscal policy ***has* to be Ricardian even when monetary policy is passive**

Standard Result		
	Fiscal Policy is	
	Ricardian	Non-Ricardian
Taylor holds	Determinacy	No equilibrium
does not hold	Multiplicity	Determinacy

With Our Perturbation		
	Fiscal Policy is	
	Ricardian	Non-Ricardian
Taylor holds	Determinacy	No equilibrium
does not hold	Determinacy	No equilibrium

Feedback Rules and the Ramsey Implementation

- Consider the **Ramsey optimum**. How can monetary policy uniquely implement it?
- If the monetary authority **observes the underlying shocks**, uniquely implemented with:

$$i_t = i_t^o + \phi(\pi_t - \pi_t^o),$$

  where i_t^o and π_t^o are rates and inflation in the optimum and $\phi > 1$.

- What if the monetary authority **does not observe the underlying shocks?**
  - implemented through feedback rules?

$$i_t = \phi \pi_t$$

- Two conflicting roles
  - **Stabilization** ($\phi < 1$ possible in the Ramsey optimum)
  - **Eq. selection** ($\phi > 1$ necessary in the standard paradigm)
- Here: Liberates the **stabilization role** of monetary policy from **its eq. selection role**

Local vs Global Determinacy

- How to read our results in a more general nonlinear environment?
- Guarantee **local determinacy**
  - linearize around a steady state & focus on bounded eq.
  - local determinacy *regardless* of the eigenvalues
- Do not speak to **global determinacy**
  - allow multiple steady state equilibria
  - our methods presume common knowledge of any given steady state

The Flexible Price Case

- Result valid for any value of κ (the slope of NKPC).
- What if prices are *literally* flexible, or "$\kappa = \infty$"
  - aggregate demand ceases to matter for aggregate output
  - economy **no longer a game among the consumers**
  - two problems **fundamentally different**
- Broader debate (Kocherlakota, 2020):
  - whether flexible-price models are proper limits of models with nominal rigidity

Alternative Boundedly-Rational Solution Concepts

1. **Relax REE but maintain a "fixed point" between expectations & actual eq.**
    - e.g., Cognitive discounting in Gabaix (20); Diagnostic expectations in Bordalo et. al (20)
    - may shrink the determinacy region but the indeterminacy problem remains
2. Completely **shuts down the "fixed point"**
    - e.g. Level-k thinking (Garcia-Schmidt & Woodford, 19; Farhi & Werning, 19)
    - produces a unique solution but opens a new issue
    - whenever $\phi < 1$, **Level-k solution becomes infinitely sensitive to Level-0 behavior**

Outline

Take-home Messages

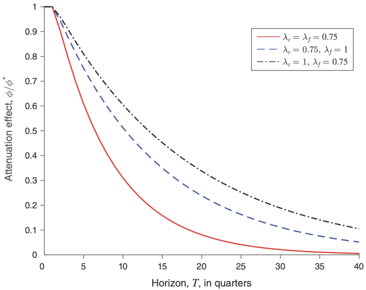
- Warning: as in global games, multiplicity can strike back with enough CK
- Still, our results
  - **illustrate fragility of sunspot/backward looking solutions**
  - **help escape the equilibrium selection conundrum**
- A new perspective on **both** the **Taylor principle** and the **FTPL**
  - Recast Taylor principle as stabilization instead eq. selection
  - Reformulate FTPL outside the equilibrium selection logic
    e.g., model MP-FP interaction as a game between monetary and fiscal authority

Bonus: Revisiting the MSV Solution

- So far: θ_t is commonly known at $t \Rightarrow$ **MSV solution c_t^F unaffected by friction**
- How does c_t^F change if θ_t is **not commonly known**?
- That's the question studied by existing literature
  - Woodford, Mankiw-Reis, Mackowiac-Wiederholt
  - my earlier work with Marios
- That literature **assumes away the determinacy** issue by
  - imposing Taylor Principle
  - focuses on **"noising up" the solution** c_t^F (make it better behaved)

Illustration: Angeletos & Lian (AER 2018)

- Liquidity trap of length T, forward guidance about i_{T+1}
  - akin to a news shock at $t = 0$ for θ_{T+1}
- **Remove common knowledge about θ_{T+1}** (e.g., private signals)
- Study how effect varies with T



- Why? Because higher T maps to a large GE multiplier in the standard model, which is greatly **attenuated by lack of common knowledge**

Conclusion (2): Two Birds with One Stone

- Common theme:
  - coordination friction **attenuates strategic complementarity/GE feedback**
- This paper:
  - **resolve the classic indeterminacy issue**
  - reinforce logical foundations of the MSV solution
- The earlier paper:
  - **attenuate the aggregate response to macro news**
  - resolves the forward guidance puzzle