Borrowing Constraints, Markups, and Misallocation*

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Abstract

We document that less constrained firms in an industry have higher markups. This connection between markups and borrowing constraints has important allocative efficiency implications: it lowers the TFP losses from markup dispersion, because looser borrowing constraints help higher markup firms produce more and move their market shares closer to the efficient level. We further document that this relationship is stronger in industries where firms rely more on earnings to borrow, and that markup dispersion is also higher in these industries. We explain all of these patterns using a standard Kimball demand model augmented with borrowing against assets and earnings, and show that the connection between markups and borrowing constraints substantially lowers TFP losses from markup dispersion, especially when firms rely on earnings to borrow.

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1 Introduction

A growing literature has provided key insights into how markup dispersion contributes to misallocation (Peters, 2020; Edmond, Midrigan, and Xu, 2023; Baqaee, Farhi, and Sangani, 2024a,b). In these studies, variations in firms' markups are driven by differences in productivity, and markup dispersion leads to substantial TFP losses. Firms' production activities and markups, however, can also depend on their ability to raise financing, such as constraints on borrowing emphasized in macro-finance research (Bernanke, Gertler, and Gilchrist, 1999; Ottonello and Winberry, 2020).

In this paper, we present empirical and theoretical analyses showing that variations in firms' markups are linked to differences in the tightness of their borrowing constraints, with less constrained firms having higher markups. This link between markups and borrowing constraints has important allocative efficiency implications: it substantially reduces the TFP losses from markup dispersion. Because higher markup firms face looser borrowing constraints, they can produce more, moving their market shares closer to the efficient level. This effect is especially pronounced when firms rely on earnings to borrow.

We first provide new empirical evidence that links firms' markups with their borrowing constraints, using both U.S. Compustat data and Census of Manufactures data. We start by documenting that, within an industry, less constrained firms have higher markups (Fact 1a). For markups, we use several estimates from De Loecker, Eeckhout, and Unger (2020) in Compustat data and the measure based on the ratio of sales over intermediate inputs plus wage bill in Census data, as well as translog markups following De Ridder (2024) in both datasets. For constraint tightness, we use cash holdings (normalized by book assets) in Compustat data, which is a common approach in the literature (Gilchrist et al., 2017): firms with abundant cash holdings can use internal funds and are less constrained, while those with little cash need to rely on external borrowing and are more constrained. We also consider alternative measures of constraint tightness, such as the distance to violating financial covenants in debt contracts and the log capital wedge, based on log production worker hours over capital in Census data, a measure which aligns with our model.

Furthermore, we find that the relationship between markups and constraint tightness is stronger in industries where firms have low liquidation values of pledgeable assets and therefore rely more on earnings to borrow (Fact 1b). Markup dispersion is also higher in these industries that rely more on earnings to borrow (Fact 2). These industry-level relationships hold in both Compustat data and Census of Manufactures data.

We show how these firm-level and industry-level results can be quantitatively explained by a standard Kimball demand model augmented with borrowing against assets and earnings. In this model, the competitive final good producer aggregates differentiated intermediate goods through a Kimball aggregator. Monopolistically competitive intermediate goods producers use labor and capital and face asset-based and earnings-based borrowing constraints. Specifically, firms can pledge assets on a standalone basis and borrow up to the liquidation value of the assets (asset-based borrowing constraints), or pledge the earnings from their operations and borrow up to a multiple of the earnings (earnings-based borrowing constraints). The overall debt limit is the greater value of the two options. Earnings-based borrowing constraints are affected by firms' productivity and markups, whereas asset-based borrowing constraints are driven by how generic the production assets are, which varies across industries due to the production assets' physical characteristics as shown by Kermani and Ma (2023a). Intermediate good producers not only optimally choose the amount of labor and capital used for production but also endogenously choose their saving.

Two key mechanisms in the model explain the negative correlation between constraint tightness and markups. First, looser borrowing constraints lower marginal costs, allowing firms to attain larger market shares and, because of Kimball demand, charge higher markups. This applies in general for both asset-based and earnings-based borrowing constraints. Second, when firms rely on earnings to borrow, higher productivity firms tend to have looser constraints because of their higher earnings. These firms also tend to have higher markups, again because of their larger market shares and Kimball demand. This further generates a negative correlation between constraint tightness and markups. This mechanism is, however, absent under asset-based borrowing constraints, because higher productivity firms instead face tighter constraints due to their greater borrowing needs, and their higher earnings and markups do not help relax asset-based constraints. Both mechanisms explain Fact 1a. Because the second mechanism operates only under earnings-based borrowing constraints, the negative relationship between markups and constraint tightness is stronger in industries that rely more heavily on earnings to borrow, explaining Fact 1b. In turn, since higher markup firms in these industries face looser borrowing constraints, they can borrow more and further expand their market shares, raising their markups even higher and increasing overall markup dispersion, explaining Fact 2.

The interaction between markups and borrowing constraints has important allocative efficiency implications: in particular, it reduces the overall TFP losses from markup dispersion. To illustrate further, we rewrite the Hsieh and Klenow (2009) formula for TFP losses (extending it to the case with Kimball demand) to express the TFP losses as an increasing function of the variance in log markup and constraint tightness (measured in log capital wedge, defined as in Hsieh and Klenow (2009)), along with the covariance between log markup and constraint tightness.

$$\begin{aligned}\text{Total TFP loss} \quad \approx \quad & \underbrace{\text{log markup dispersion}}_{>0} \quad + \quad \text{dispersion in borrowing constraint tightness} \\ & + \underbrace{\text{covariance of log markup \& borrowing constraint tightness}}_{<0 \text{ if higher markup firms less constrained}} \qquad (1)\end{aligned}$$

This formula illustrates how introducing borrowing constraints mitigates the TFP loss from markup dispersion. Without borrowing constraints, markup dispersion leads to TFP losses, as summarized by the first term in (1). With borrowing constraints, the negative last term in the TFP formula (1) mitigates the TFP losses from markup dispersion because high markup firms are less constrained (Fact 1a), as explained by the two mechanisms above. When high markup firms face looser constraints, they produce more and their market share moves closer to the planner's counterpart, thereby reducing TFP losses. Furthermore, how much the borrowing constraint channel mitigates TFP losses from markup dispersion also varies across industries, given the empirical findings in Fact 1b and Fact 2. In industries that rely more on earnings to borrow, markup dispersion is higher, but markups are also more negatively correlated with constraint tightness. Indeed, in our model, the latter channel dominates, and the net TFP loss from markup dispersion (after accounting for this negative covariance) is smaller in those industries.

Finally, the TFP gains from subsidies designed to remove markup distortions are smaller compared to the substantial gains from such policies without borrowing constraints. The reason is that borrowing constraints already mitigate the TFP losses from markup dispersion, especially if firms rely more on earnings to borrow.

Several studies examine how firms set prices in response to *temporary negative shocks*, in the presence of financial constraints. Some find that firms with tighter constraints set higher prices (Chevalier and Scharfstein, 1995; Gilchrist et al., 2017; Meinen and Soares, 2022; Kim and Park, 2024; Renkin and Züllig, 2024). The interpretation is that firms intertemporally substitute their investment in building a customer base, and they cut back on such investment when their constraints tighten. Meanwhile, Kim (2021) and Lenzu et al. (2024) find that firms cut prices in response to credit tightening (e.g., due to banks' exposure to the Lehman crisis or the European debt crisis). Our analyses instead focus on long-run steady-state variations in markups across firms, rather than how prices respond to shocks. For example, constrained firms cannot always charge higher markups and underinvest in customer capital in the steady state (even though they can cut back in bad times and engage in intertemporal substitution). Eventually, they will lose market share (Renkin and Züllig, 2024), and correspondingly their long-term markups will fall.

Related Literature A large literature has analyzed misallocation arising from financial frictions (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014; Ottonello and Winberry, 2024) and information frictions (David, Hopenhayn, and Venkateswaran, 2016; David and Venkateswaran, 2019). Recent research offers key insights into misallocation due to markup dispersion (Peters, 2020; Baqaee and Farhi, 2020; Edmond, Midrigan, and Xu, 2023; Boar and Midrigan, 2024). The focus has been on markup dispersion due to productivity differences. An earlier version of Boar and Midrigan (2019) and Peruffo (2025) study misallocation arising from markup dispersion when firms face asset-based borrowing constraints, but do not analyze earnings-based borrowing constraints. Using both novel empirical evidence and a model that quantitatively fits these facts, we show that the possibility of borrowing against both assets and earnings generates unique interactions between firms' markups and constraint tightness, and correspondingly distinct implications for allocative efficiency. In contemporaneous work, Ebsim (2025) performs theoretical and quantitative analyses where firms charge variable markups and face only earnings-based borrowing constraints: he compares welfare losses from financial frictions with and without markup distortions and shows that for a planner constrained by financial frictions, the optimal policy does not fully eliminate the markup distortions.

The empirical and theoretical connection between markups and borrowing constraints we highlight builds on macro-finance research, especially the work on earnings-based borrowing constraints. A number of papers document that borrowing against earnings is important among U.S. nonfinancial firms, and that firms are subject to debt limits as a function of their earnings (Greenwald, 2019; Lian and Ma, 2021; Li, 2022; Drechsel, 2023; Caglio, Darst, and Kalemli-Ozcan, 2024; Adler, 2025; Zhao, 2025; Su, 2026). Such borrowing is especially prevalent when firms' pledgeable assets have low liquidation values on a standalone basis (Kermani and Ma, 2023a), in which case the traditional borrowing constraints based on asset liquidation values à la Kiyotaki and Moore (1997) are too tight and borrowing against earnings improves debt capacity if earnings are high. Borrowing against earnings leads to a close connection between markups and financial frictions.

2 Motivating Facts

In this section, we present new empirical facts at the firm level and the industry level. In Sections 3 and 4, we develop a standard Kimball demand model augmented with borrowing against assets and earnings to explain the empirical facts documented here. In Section 5, we study the implications for allocative efficiency.

2.1 Baseline Empirical Facts

In Section 2.1.1, we present baseline empirical facts using Compustat data, which cover public firms in all industries. In Section 2.1.2, we present further evidence using Census of Manufactures data, which are among the best data to measure markups in the U.S. and cover not only public companies, though this sample only covers manufacturing in Economic Census years (years ending in 2 and 7) and has limited information about firms' financial conditions.

2.1.1 Compustat Data

In the following, we lay out the key empirical facts using Compustat data.

Fact 1a. Less constrained firms have higher markups We start with the firm-level relationship between markups and constraint tightness in Panel A of Figure 1. This firm-year level binscatter plot uses the log firm-level markup from De Loecker, Eeckhout, and Unger (2020) on the y-axis, and we confirm similar results with a number of other markup estimates in Table 1. The x-axis uses a common proxy for firms' constraint tightness, namely firms' cash holdings (normalized by book assets), following Gilchrist et al. (2017) for example. This measure reflects the abundance of firms' internal funds and is inversely related to how constrained they are. The sample covers U.S. Compustat firms annually from 1987 (we start in 1987 because the Census of Manufactures data we use in Section 2.1.2 begin in 1987) to 2016 (we end in 2016 because the output elasticity estimates from De Loecker, Eeckhout, and Unger (2020) stop in 2016). We use industry (3-digit NAICS code) by year fixed effects to pinpoint the heterogeneity among firms in the same industry at a given point in time.

We perform several robustness checks for alternative measures of constraint tightness at the end of this section. First, we use the distance to violating earnings-based financial covenants in debt contracts constructed following Lian and Ma (2021). This measure links directly to borrowing limits, whereas Farre-Mensa and Ljungqvist (2016) find that firms classified as constrained according to traditional financial constraint proxies in the literature such as the Kaplan and Zingales (1997) index or the Whited and Wu (2006) index have no trouble raising debt. Second, we also check that the main empirical results hold using the log capital wedge in Census of Manufactures data, based on log production worker hours relative to capital as an alternative proxy for constraint tightness, which maps to the model in Section 3. Finally, Altomonte et al. (2024) study a policy reform in France that led to variations in firms' liquidity, and find that higher liquidity contributes to higher markups.

Figure 1, Panel A, shows a strong positive relationship between firms' markups and cash holdings. We confirm the statistical significance of this relationship in column (1) of Table 1, Panel A. This suggests that less constrained firms have higher markups. The remaining columns perform further checks using several other markup proxies. Column (2) uses markups from De Loecker, Eeckhout, and Unger (2020) where the production function includes overhead as a factor of production. Column (3) uses accounting markups from De Loecker, Eeckhout, and Unger (2020), which are the ratio of sales to cost of goods sold scaled by the industry cost share. Column (4) uses translog markups following De Ridder (2024). We exclude observations with markup less than one, which would be abnormal and could arise from measurement issues (this issue seems more severe for accounting markups). We use industry (3-digit NAICS code) by year fixed effects, and double cluster standard errors by both industry and year. The magnitude of the regression coefficient implies that a one standard deviation change in cash over assets is associated with about a 0.2 standard deviation change in firm markups. In Table IA1, we show that the results are robust to controlling for firm age. Since Compustat does not have a direct measure of firm age, we use the older of the year since incorporation (from Refinitiv) and the year since IPO (from Compustat). Indeed, the coefficients on cash holdings in Table IA1 are very similar to those in Table 1.

One concern is that the revenue-based markup estimates in De Loecker, Eeckhout, and Unger (2020) are not informative of the level of markups (Bond et al., 2021). As De Ridder, Grassi, and Morzenti (2026) point out, although the average level of markups can be affected by biases, these measures still capture the differences in markups across firms and hence markup dispersion reasonably well, which is our focus. Also, it is worth noting that our markup estimates remain valid in the presence of borrowing constraints (or, more generally, the existence of capital wedges, as pointed out by De Ridder, Grassi, and Morzenti (2026)).[1]

[1]Within the context of our model, this point can be seen from (13), which determines markups based on the intermediate good producer's optimal choice of labor. For an asset-based constraint, the borrowing limit does

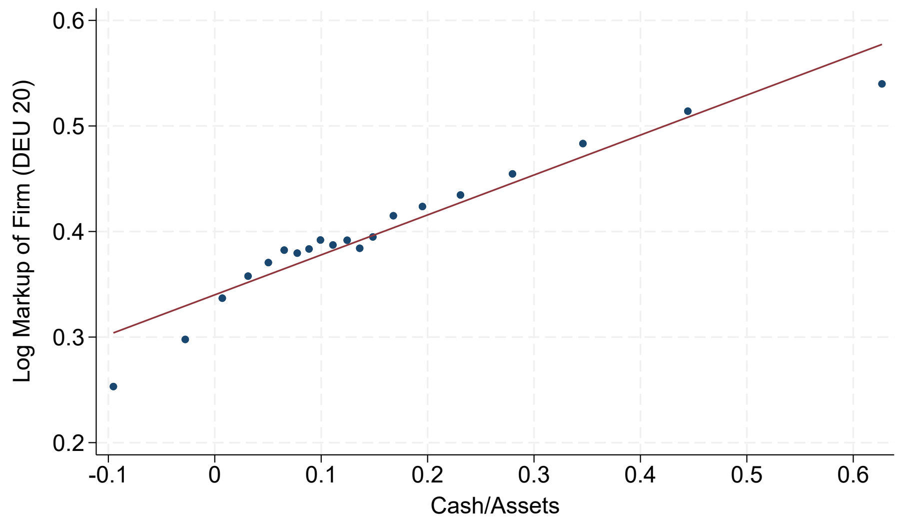
Finally, the relationship between firms' markups and constraint tightness does not need to be interpreted causally in either direction to be of interest. In our subsequent analysis, we explain how this relationship arises naturally from a standard Kimball demand model augmented with borrowing against assets and earnings. Moreover, their univariate relationship (without controls) is of particular interest, as the univariate covariance between markup and constraint tightness directly enters the TFP loss (26) and determines the allocative efficiency implications.

Fact 1b. The relationship between markups and constraint tightness is stronger in industries that rely more on earnings to borrow Furthermore, we find that the relationship between markups and constraint tightness is stronger among industries where firms have low liquidation values of pledgeable assets and therefore rely more on borrowing against earnings (Lian and Ma, 2021; Kermani and Ma, 2023a). In the binscatter plot in Figure 1, Panel B, we study the covariance between firm-level log markup and cash over assets (i.e., the covariance of the two variables on the two axes in Panel A) in each industry-year on the y-axis. We use the industry-level liquidation value of pledgeable assets such as property, plant, and equipment and working capital (normalized by book assets) from Kermani and Ma (2023a,b) on the x-axis, which is driven by physical characteristics of assets used in each industry and shapes the extent to which firms need to borrow against earnings.[2] When the liquidation value of pledgeable assets is high, firms can directly pledge them for asset-based borrowing; when the liquidation value is low, firms rely more on earnings to borrow, which we verify in Appendix Figure IA1. Figure 1, Panel B, shows that the relationship between markups and cash over assets in Panel A is especially strong in low liquidation value industries where firms rely more on earnings to borrow. We confirm the statistical significance of this relationship in column (1) of Table 1, Panel B. Columns (2) to (4) use other markup measures (for the covariance between log markup and cash holdings on the left hand side) parallel to those in Panel A. We use year fixed effects, and double cluster standard errors by industry and year. The magnitude of the regression coefficient implies that a one standard deviation change in the industry-level liquidation value is

not depend on labor. For an earnings-based constraint, the borrowing limit depends on labor through earnings, but the Lagrange multiplier on the borrowing constraint rescales marginal revenue and the variable labor cost proportionally, so it cancels from the labor first-order condition. Hence borrowing constraints do not alter (13).

[2]We follow Kermani and Ma (2023b) and calculate the asset liquidation value of Compustat firms in industry k and year t as: $\text{LiqVal}_{kt} = \sum_j \lambda_k^j A_{kt}^j$, where A_{kt}^j is the book value of asset type j (including property, plant, and equipment, inventory, and receivables, which are commonly pledged for asset-based borrowing) of Compustat firms in industry k and year t, and λ_k^j is the liquidation recovery rate (liquidation value per dollar of book value) for asset type j in industry k from Kermani and Ma (2023a). We then normalize LiqVal_{kt} by total book assets of Compustat firms in industry k and year t.

Panel A. Less Constrained Firms Have Higher Markups



Panel B. Relationship Stronger in Industries that Rely More on Earnings to Borrow

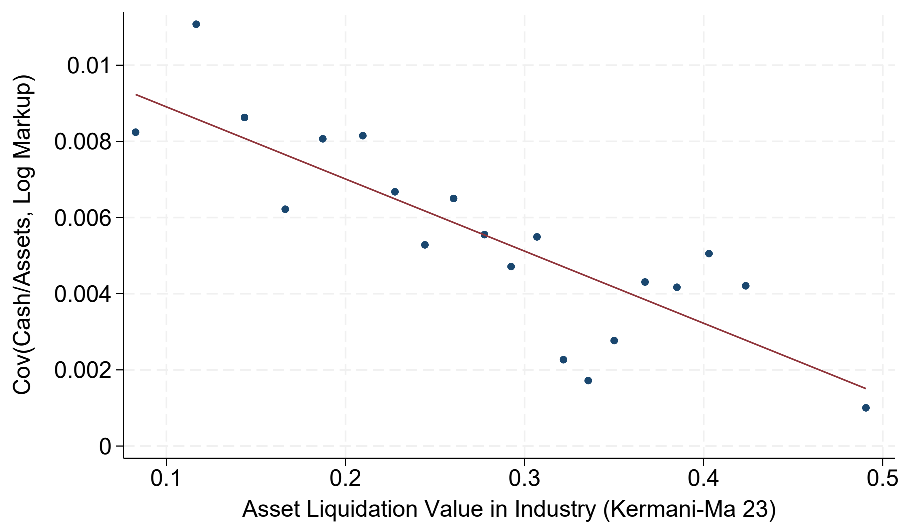


Figure 1: Firm Markups and Constraint Tightness. Panel A shows a binscatter plot of firm-level log markup on the y-axis and firm-level cash holdings (normalized by book assets) on the x-axis, using U.S. Compustat firms annually from 1987 to 2016. Markups use the baseline measure in De Loecker, Eeckhout, and Unger (2020). Industry (3-digit NAICS code) by year fixed effects are included. Panel B shows the covariance between firm-level log markup and cash/assets in each industry-year on the y-axis, and the industry's liquidation value of fixed assets plus working capital (normalized by book assets) on the x-axis. The liquidation value data are from Kermani and Ma (2023a). Year fixed effects are included. We exclude observations with markup less than one.

associated with about a 0.1 standard deviation change in the covariance between firm markups and cash over assets in the industry.

Similar regressions in Table IA3, Panel A, show that the positive covariance between markups and cash over assets is also stronger when the industry has a larger share of borrowing against earnings (cash flow-based debt) in its total debt. The magnitude of the regression coefficient

Table 1: Firm-Level Markups and Constraint Tightness (Compustat)

Panel A. Less Constrained Firms Have Higher Markups

	Firm Log Markup			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Cash/Assets	0.378***	0.333***	0.192***	0.304***
	(0.036)	(0.030)	(0.014)	(0.048)
Fixed Effects		Year		
Observations	121,789	80,937	60,174	95,300
R^2	0.29	0.26	0.19	0.29

Panel B. Relationship Stronger in Industries that Rely More on Earnings to Borrow

	Industry Cov(Log Markup, Cash/Assets)			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.019***	-0.012*	-0.009**	-0.013***
	(0.006)	(0.007)	(0.004)	(0.005)
Fixed Effects		Year		
Observations	2,351	2,223	2,268	2,262
R^2	0.05	0.03	0.02	0.03

Notes. Panel A shows firm-year level regressions Log Markup$_{it} = \alpha_{ind(i)t} + \beta$Cash/Assets$_{it} + \varepsilon_{it}$, using U.S. Compustat firms annually from 1987 to 2016. Log Markup$_{it}$ for firm i in year t uses the baseline markup from De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input from De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup from De Loecker, Eeckhout, and Unger (2020) in column (3), and translog markup following De Ridder (2024) in column (4). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Panel B shows industry-year level regressions Cov(Log Markup, Cash/Assets)$_{kt}$ = $\alpha_t + \beta$Liqval$_{kt} + \varepsilon_{kt}$, among U.S. Compustat firms. Cov(Log Markup, Cash/Assets)$_{kt}$ is the covariance of firm-level log markup and cash/assets among firms in industry k in year t. The set of markups is the same as the columns in Panel A. The variable Liqval$_{kt}$ is the industry's asset liquidation value from fixed assets and working capital normalized by the industry's total book assets. The liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a). Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

implies that a one standard deviation change in the share of cash flow-based debt in the industry is associated with about a 0.1 standard deviation change in the covariance between firm markups and constraint tightness. This magnitude is comparable to that in Table 1, Panel B discussed above.

Fact 2. Markup dispersion is higher in industries that rely more on earnings to borrow Next, we show that markup dispersion is higher in industries with low liquidation values of pledgeable assets and correspondingly a greater reliance on borrowing against earnings. The binscat-

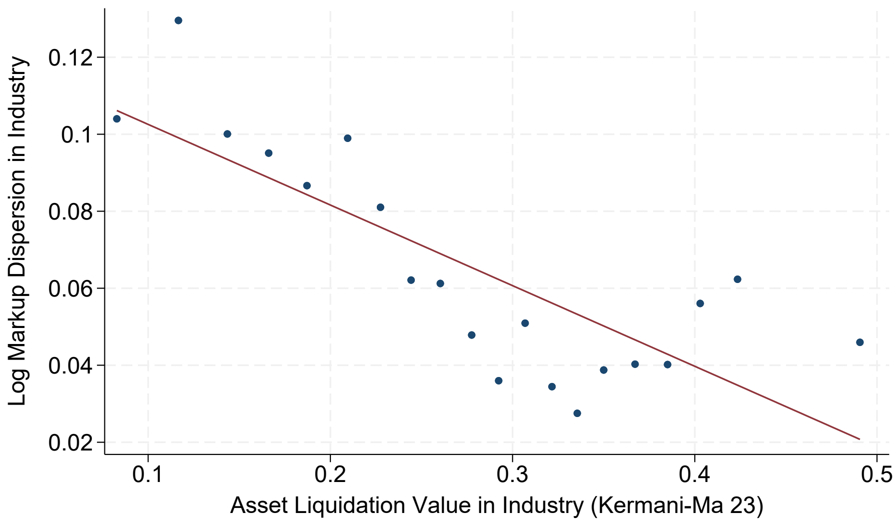


Figure 2: Industry Markup Dispersion (Compustat). This figure shows a binscatter plot of the relationship between the variance of firm-level log markup in each industry-year on the y-axis, and the industry's liquidation value of fixed assets plus working capital (normalized by book assets) on the x-axis, using Compustat firms annually from 1987 to 2016. Markups use the baseline measure from De Loecker, Eeckhout, and Unger (2020). The liquidation value data are from Kermani and Ma (2023a). Year fixed effects are included. We exclude observations with markup less than one.

ter plot in Figure 2 shows the variance of firm-level log markup within an industry-year on the y-axis, and the industry's liquidation value of pledgeable assets on the x-axis (same x-axis as Figure 1, Panel B). We confirm the statistical significance of this relationship in column (1) of Table 2. Columns (2) to (4) use other markup measures parallel to those in Table 1. The magnitude of the regression coefficient implies that a one standard deviation change in the industry-level liquidation value is associated with about a 0.3 standard deviation change in the industry's markup dispersion. Similar regressions in Table IA3, Panel B, show that markup dispersion is also higher when the industry has a larger share of borrowing against earnings in total debt. The magnitude of the regression coefficient implies that a one standard deviation change in the share of cash flow-based debt in the industry is associated with about a 0.2 standard deviation change in the industry's markup dispersion. This magnitude is comparable to that in Table 2.

2.1.2 Census of Manufactures Data

In this section, we examine the key empirical facts using Census of Manufactures, which is one of the best datasets for markup measurement in the U.S. (Autor et al., 2020). In this dataset, we cover a comprehensive set of firms in the economy (not just public companies), but the data are restricted to manufacturing firms in Economic Census years (years ending in 2 and 7).[3]

[3]We use the same cleaning procedure as the Dispersion Statistics on Productivity (DiSP) database from the Census Bureau (U.S. Census Bureau, 2024), and the sample runs from 1987 to 2017.

Table 2: Industry Markup Dispersion (Compustat)

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.260***	-0.249***	-0.222***	-0.243***
	(0.066)	(0.064)	(0.052)	(0.055)
Fixed Effects		Year		
Observations	3,820	3,578	3,923	3,006
R^2	0.13	0.10	0.10	0.14

Notes. This table shows industry-year level regressions $\text{Var(Log Markup)}_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, using U.S. Compustat firms annually from 1987 to 2016. $\text{Var(Log Markup)}_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Table 1. The variable Liqval_{kt} is the industry's liquidation value from fixed assets and working capital normalized by the industry's total book assets. The liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a). Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

In this dataset, we construct a markup measure using the ratio of sales to the sum of intermediate inputs and wage bill. This ratio only differs by a scalar from the standard markup measurement such as the ratio estimator in De Loecker, Eeckhout, and Unger (2020), which captures the output elasticity. Because we focus on cross-firm differences in log markup and on the dispersion of log markup, the scalar is irrelevant for our analysis.[4] We also construct translog markups following De Ridder (2024). We verify that markups in Census and Compustat data (using the De Loecker, Eeckhout, and Unger (2020) approach) are significantly positively correlated, as shown in Table IA6.

Since financial information like cash is not available in census data, we construct an alternative measure of constraint tightness that is feasible in this dataset. Specifically, we follow the model in Section 3 and use the log capital wedge, based on the log of production worker hours relative to capital.[5] A high capital wedge indicates constraints as firms cannot acquire enough capital, so they use "too little" capital relative to labor. Although capital intensity may differ across industries for other reasons, our analyses only use variations of the capital wedge among firms in the same industry-year.

[4] This approach is valid if the output elasticity is homogenous within an industry. The translog markup measure relaxes this assumption and demonstrates the robustness of our findings.

[5] Again, the ratio of production worker hours to capital differs from its theoretical counterpart in the definition of capital wedge in (14) by a scalar irrelevant for our purposes.

Fact 1a. Less constrained firms have higher markups In Table 3, Panel A, we examine the firm-level relationship between log markup and log capital wedge, the measure of constraint tightness in census data. We find that firms with higher capital wedge (more constrained) have significantly lower markups. This pattern is consistent with results in Table 1 (the results using log capital wedge in Table 3 have the opposite sign as those using cash holdings in Table 1 because higher capital wedges indicate tighter constraints whereas higher cash holdings indicate looser constraints). We include industry by year fixed effects as in Table 1, so the relationship between markups and capital wedges here reveals within-industry variations at a given point in time.

Fact 1b. The relationship between markups and constraint tightness is stronger in industries that rely more on earnings to borrow In Table 3, Panel B, we show that the above relationship is stronger for industries with low asset liquidation values. We regress the covariance of log markup and log capital wedge on the industry asset liquidation value, and find that the covariance is more negative when asset liquidation values are low (where firms need to rely more on earnings to borrow). Here we merge the industry-level asset liquidation value measure used in Section 2.1.1 with the Census data (based on 3-digit NAICS code), assuming that the liquidation value is an industry characteristic (any measurement noise would work against us finding significant results).

Fact 2. Markup dispersion is higher in industries that rely more on earnings to borrow Finally, Table 4 uses the Census data to verify that markup dispersion (measured using the variance of log firm-level markups within an industry-year) is smaller in industries with high asset liquidation values. Figure IA3 presents a binscatter plot to display this relationship visually.

2.2 Further Discussions

2.2.1 Alternative Proxies of Constraint Tightness

As mentioned above, we use alternative measures of constraint tightness to perform robustness checks for the main results. First, we use the distance to violating earnings-based financial covenants in debt contracts, which is directly linked to how binding debt limits are: these financial covenants in corporate loans impose limits on the maximum total debt to EBITDA (earnings before interest, taxes, depreciation, and amortization) ratio or the minimum EBITDA to debt payments ratio, and firms need to show compliance on a quarterly basis. We calculate

Table 3: Firm-Level Markups and Constraint Tightness (Census of Manufactures)

Panel A. Less Constrained Firms Have Higher Markups

	Firm Log Markup	
	(1)	(2)
Markup Measure	Baseline	Translog
Log Capital Wedge	-0.014***	-0.010***
	(0.003)	(0.003)
Fixed Effects	Industry × Year	
Observations	164,000	164,000
R^2	0.10	0.06

Panel B. Relationship Stronger in Industries that Rely More on Earnings to Borrow

	Industry Cov(Log Markup, Log Capital Wedge)	
	(1)	(2)
Markup Measure	Baseline	Translog
Industry Asset Liquidation Value	0.101***	0.079***
	(0.019)	(0.017)
Fixed Effects	Year	
Observations	150	150
R^2	0.28	0.23

Notes. Panel A shows firm-year level regressions Log Markup$_{it} = \alpha_{ind(i)t} + \beta$Log Capital Wedge$_{it} + \varepsilon_{it}$, using Census of Manufactures firms in Economic Census years from 1987 to 2017. Log Markup$_{it}$ for firm i in year t uses the log of sales over intermediate inputs plus wage bill in column (1) and log translog markup following De Ridder (2024) in column (2). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Panel B shows industry-year level regressions Cov(Log Markup, Log Capital Wedge)$_{kt} = \alpha_t + \beta$Liqval$_{kt} + \varepsilon_{kt}$, among Census of Manufactures firms in Economic Census years. Cov(Log Markup, Log Capital Wedge)$_{kt}$ is the covariance of firm-level log markup and log capital wedge among firms in industry k in year t. The set of markups is the same as the columns in Panel A. The variable Liqval$_{kt}$ is the industry-level asset liquidation value from Table 1, Panel B. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

the overall distance to the most binding limit in outstanding loans, following the construction in Lian and Ma (2021). In Figure IA2 and Table IA2, we show that markups are higher among firms that are further from these covenants being binding. This result aligns with Fact 1a, which finds that less constrained firms have higher markups. Since this measure is only available for firms with loans that include financial covenants, it is less suitable for the robustness check for Fact 1b, due to both selection and the limited number of observations in each industry-year.

Second, in our model in Section 3, the constraint tightness will be reflected by the capital wedge. In Section 2.1.2, we measure the log capital wedge in the Census data based on the log of production worker hours relative to capital. We observe consistent results using this measure as well.

Table 4: Industry Markup Dispersion (Census of Manufactures)

	Variance of Firm-Level Log Markup in Industry	
	(1)	(2)
Markup Measure	Baseline	Translog
Industry Asset Liquidation Value	-0.093*	-0.091*
	(0.041)	(0.043)
Fixed Effects	Year	
Observations	150	150
R^2	0.30	0.29

Notes. This table shows industry-year level regressions $\text{Var(Log Markup)}_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, using Census of Manufactures firms in Economic Census years from 1987 to 2017. $\text{Var(Log Markup)}_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Table 3. The variable Liqval_{kt} is the industry-level asset liquidation value from Table 2. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

2.2.2 Relevance of Earnings-Based Borrowing Constraints

Our analyses in this paper highlight that earnings-based borrowing constraints can shape variations in markups documented above. A common question is whether companies such as those in Compustat are constrained. Lian and Ma (2021) provide an extensive study of the relevance of earnings-based borrowing constraints among large U.S. nonfinancial firms. They find that almost 70% of large Compustat nonfinancial firms have earnings-based borrowing constraints written explicitly in their debt contracts. In a given year, about 10% of these firms violate the constraints and 25% are within one standard deviation to violation. This importance of earnings-based borrowing constraints is frequently discussed by firms' financial reports. For example, Starwood Hotel's 2011 annual report states: "A substantial decrease in EBITDA ... could make it difficult for us to meet our debt service requirements and restrictive covenants and force us to sell assets and/or modify our operations." GNC's 2012 annual report states: "These restrictions may prevent us from taking actions that we believe would be in the best interest of our business and may make it difficult for us to successfully execute our business strategy or effectively compete with companies that are not similarly restricted."

2.2.3 Covariates with Industry Asset Liquidation Values

One question is whether the asset liquidation value used in Fact 1b and Fact 2 might be correlated with other industry characteristics that can affect markup dispersion. In particular, it is important to check that our results are not due to asset liquidation value being correlated with the volatility of shocks in an industry, which can contribute to markup dispersion as well. First, we follow the literature and capture the volatility of shocks using stock return volatility, measured as the average firm-level stock return volatility in the industry in a given year. Table IA4 presents the results. The coefficients on asset liquidation value are very similar to those in the baseline regressions in Table 2. Second, our model in Section 3 will show that the dispersion of earnings to debt in an industry is an important reflection of the volatility of shocks, and we control for it in Table IA5 (we use earnings to debt instead of debt to earnings because firms with low earnings may have an arbitrarily large debt-to-earnings ratio that adds noise to the measurement). The coefficients on asset liquidation value are again very similar to those in the baseline regressions in Table 2.

The asset liquidation value, by construction, is also related to the degree of investment irreversibility: Can irreversibility affect markup dispersion in other ways, such as investment adjustment costs? However, this channel does not naturally imply a relationship between markups and constraint tightness, and it is not obvious that it makes such a relationship significantly stronger when industry asset liquidation values are low. Another question is how to think about intangible assets: low asset liquidation values may be correlated with having a large amount of intangible assets in an industry. To the extent that intangible assets (e.g., organizational capital) can be difficult to pledge for asset-based borrowing, so more intangible assets make firms more reliant on pledging earnings for borrowing, then the same mechanisms apply.

3 Model

In this section, we set up an industry equilibrium model featuring a standard Kimball demand aggregator, augmented to allow borrowing against both assets and earnings. Later, we show that it explains the firm-level and industry-level evidence in Section 2, and use it to study implications for TFP losses from markup dispersion.

In this model, the industry's final good is produced by competitive firms that aggregate differentiated intermediate goods through a Kimball aggregator.[6] Monopolistically competitive intermediate good producers use labor and capital and are subject to asset-based and earnings-based borrowing constraints.

[6]Our mechanism also applies when markup dispersion arises from oligopolistic competition with nested-CES demand, instead of Kimball demand, as in Atkeson and Burstein (2008). Edmond, Midrigan, and Xu (2023) shows the impact of markup dispersion on misallocation is similar under the two market structures.

Industry final good producers The industry's final good is produced by perfectly competitive producers that aggregate a continuum of intermediate goods $\{y(i)\}_{i\in[0,1]}$ into the final output Y via a Kimball aggregator $\Upsilon(\cdot)$:

$$\max_{\{y(i),Y\}_{i\in[0,1]}} PY - \int p(i)\, y(i)\, di \quad \text{s.t.} \quad \int \Upsilon\left(\frac{y(i)}{Y}\right) di \;=\; 1, \tag{2}$$

where $\Upsilon(1) = 1$, $\Upsilon'(\cdot) > 0$, $\Upsilon''(\cdot) < 0$, and P captures the price of the industry final good. Optimality by final good producers implies that the market share of each intermediate good i, $\tilde{y}(i) = y(i)/Y$, is related to its price $p(i)$ by

$$p(i) = p\left(\tilde{y}(i)\right) \equiv \frac{\lambda}{Y}\Upsilon'\left(\tilde{y}(i)\right), \tag{3}$$

where λ captures the Lagrange multiplier on the constraint in (2). Relationship (3) is the demand curve faced by the intermediate good producer i.

When illustrating key mechanisms of the model and how it quantitatively explains the empirical evidence in the previous section, we use a popular specification of $\Upsilon(\cdot)$ based on Dotsey and King (2005) for the Kimball aggregator,[7]

$$\Upsilon\left(\tilde{y}(i)\right) = \frac{\sigma}{\sigma\left(1-\eta\right)-1}\left(\left[\left(1-\eta\right)\tilde{y}(i)+\eta\right]^{1-\frac{1}{\sigma(1-\eta)}} - 1\right) + 1, \tag{4}$$

which implies the demand elasticity

$$\sigma\left(\tilde{y}(i)\right) \equiv -\frac{\partial \ln \tilde{y}(i)}{\partial \ln p(i)} = \sigma\left(1+\eta\frac{1-\tilde{y}(i)}{\tilde{y}(i)}\right), \tag{5}$$

where $\sigma > 1$ governs the average demand elasticity and $\eta \geq 0$ governs the "superelasticity," i.e., how the demand elasticity of good i decreases with its market share $\tilde{y}(i)$. When $\eta = 0$, the demand system corresponds to the standard Constant Elasticity of Substitution (CES) case. Note that this specification is closely related to the specification in Klenow and Willis (2016) and, in fact, shares the exact same first-order approximation with Klenow and Willis (2016). Due to differences in higher-order terms, Dotsey and King (2005)'s specification generates greater markup dispersion and allows the model to better match the data, facilitating our quantitative analysis. We henceforth use Dotsey and King (2005)'s specification throughout the paper. But the economic lessons we develop here apply to the general formulation of the Kimball aggregator.

[7]We rewrite the specification $\Upsilon(\cdot)$ in Dotsey and King (2005) to facilitate parameter interpretation: η here corresponds to $-\eta_o$, where η_o is the original η in Dotsey and King (2005), and σ here corresponds to $\frac{1}{(1+\eta_o)(1-\gamma_o)}$ in the original specification, where γ_o is the original γ in Dotsey and King (2005).

Industry intermediate good producers Each industry is populated by a unit measure of intermediate good producers $i \in [0,1]$ operating under monopolistic competition. We write the intermediate good producer's optimization problem recursively and omit the index i for simplicity. The relevant state variables are the pre-production net worth a and the productivity z. Each intermediate good producer produces a differentiated good using a Cobb-Douglas technology in capital k and labor l[8]

$$y = zk^{\alpha} l^{1-\alpha}, \tag{6}$$

where $\alpha \in (0,1)$. Each producer faces idiosyncratic productivity shocks. Its productivity, denoted by z, evolves exogenously according to

$$\log z' = \rho \log z + \epsilon_z, \tag{7}$$

where $\rho \in [0,1)$ and $\epsilon_z \sim \mathcal{N}\left(0, \sigma_e^2\right)$. The idiosyncratic productivity shock, ϵ_z, is independent across time and across firms.

Following Moll (2014), the producer can borrow $b \equiv k - a$ to acquire capital beyond its net worth for production, and the price of capital is normalized to one. Specifically, the producer chooses its capital k and labor l to maximize its post-production net worth

$$n(a,z) \equiv \max_{k,l} \quad p\left(y/Y\right) y - wl - c - (k-a)(1+r) + k(1-\delta), \tag{8}$$

subject to the demand for its goods in (3), the production technology (6), and asset-based and earnings-based borrowing constraints:

$$b \equiv k - a \leq \max\left\{\phi^A k, \phi^{\pi} \pi\right\}, \tag{9}$$

where w is the wage, c is the fixed operating cost, r is the interest rate, δ is the depreciation rate of capital, ϕ^A is the capacity for borrowing against assets (which corresponds to the industry asset liquidation value behind Fact 1b and Fact 2 in Section 2), ϕ^{π} is the capacity for borrowing against earnings, and

$$\pi \equiv p\left(y/Y\right) \cdot y - wl - c \tag{10}$$

is the producer's earnings.

The producer's overall borrowing capacity in (9) is based on contractual evidence from Lian and Ma (2021). It is determined by the maximum from borrowing against assets, as in Moll

[8]We interpret our model as a model for value added production, but the main analysis and results can be easily extended to a model for gross production in which materials inputs m are explicitly modeled and the firm's production is given by $y = z\left(k^{\alpha} l^{1-\alpha}\right)^{\gamma} m^{1-\gamma}$.

(2014), and borrowing against earnings, as in Lian and Ma (2021). Specifically, the firm can pledge assets on a standalone basis and borrow up to the liquidation value of the assets (asset-based borrowing constraints), which is the value of the firm's assets if it is liquidated upon default and the assets are sold off piecemeal. This liquidation value determines the capacity ϕ^A and varies across industries due to differences in the assets' physical characteristics such as transportation costs, customization, and durability as shown by Kermani and Ma (2023a). Alternatively, the firm can pledge earnings from its operations and borrow up to a multiple of the earnings (earnings-based borrowing constraints), which approximates the value from the firm's operations if it undergoes restructuring upon default and continues production. This multiple determines the capacity ϕ^π and is similar across industries (Lian and Ma, 2021).[9]

After production, the producer decides how much dividend d to pay and how much to save a'. Specifically, the producer chooses a' to maximize its value

$$V(a,z) = \max_{a' \geq 0} \quad d + \beta(1-x)\mathbb{E}_z\left[V\left(a',z'\right) \mid z\right] + \beta x a' \tag{11}$$

subject to the budget constraint

$$\psi_d d^2 \mathbb{I}(d<0) + d = n(a,z) - a', \tag{12}$$

where $n(a,z)$ is its post-production net worth in (8), x is the exogenous exit rate, and ψ_d is the cost of equity issuance (namely a negative dividend payment). Exiting firms are replaced by entrants with an initial net worth a^{ent} and productivity $\log z^{ent} = \mathbb{E}\left[\log z\right]$.

The model closely follows Moll (2014), particularly its discrete time version detailed in Appendix G, with only three key differences. First, the producer is monopolistically competitive rather than perfectly competitive and faces the demand curve in (3). Second, the producer's borrowing capacity in (9) is determined by the maximum implied by asset-based and earnings-based borrowing constraints, rather than by asset-based borrowing constraints alone. Third, the model does not treat the producer as an entrepreneur with log utility; instead, the producer's value is linear in its dividend payment, and the producer is subject to costly equity issuance and an exogenous probability of exit. The latter assumptions closely follow Ottonello and Winberry (2020, 2024) and better capture large firms subject to earnings-based constraints.

[9] If a firm borrows against earnings, the earnings-based constraints always restrict a firm's total debt (including both earnings-based and asset-based borrowing) as a function of its total earnings. Since the firm's assets are pledged to creditors either with their value in liquidation or with their value in continuing operation, the borrowing capacity in (9) is not given by a sum of asset-based borrowing constraints and earnings-based borrowing constraints.

Given the exogenous interest rate r, the wage w, and the demand for the industry final good D, an industry equilibrium consists of a collection of intermediate good producers' decision rules, prices, and quantities $\{a'(a,z), b(a,z), d(a,z), y(a,z), l(a,z), k(a,z), p(a,z)\}$, the industry final good price and quantity P and Y, and the cumulative distribution function of intermediate good producers' states $G(a,z)$ such that: (i) intermediate good and final good producers optimize; (ii) the industry final good price and quantity satisfy industry demand $PY = D$, which follows naturally if the technology aggregates goods produced by different industries in a Cobb-Douglas production form, as in Hsieh and Klenow (2009); (iii) the distribution of intermediate good producers' states $G(a,z)$ is stationary given the saving policy $a'(a,z)$, the process of z in (7), entry, and exit.

4 Borrowing Constraints and Markup Dispersion

In this section, we examine the model's mechanism and performance. In Subsection 4.1, we describe the two key mechanisms that explain why less constrained firms have higher markups. In Subsection 4.2, we calibrate the model to a set of standard data moments. In Subsection 4.3, we show that these two mechanisms can quantitatively explain the empirical facts in Section 2. In Subsection 4.4, we further elaborate the mechanisms and calibration features that deliver the quantitative fit.

4.1 Why Less Constrained Firms Have Higher Markups

What drives markup differences? To explain why less constrained firms have higher markups, we first discuss what drives markup differences across intermediate good producers. In our model, markup differences depend on both intermediate good producers' productivity differences, as in Edmond, Midrigan, and Xu (2023) and Baqaee, Farhi, and Sangani (2024a,b), and on differences in the tightness of their borrowing constraints.

To see this, we first link differences in intermediate good producers' markups in our model to differences in their marginal costs. Each intermediate good producer's optimal choice of labor l implies that

$$p\left(\tilde{y}\right) = \frac{\sigma\left(\tilde{y}\right)}{\sigma\left(\tilde{y}\right) - 1} \frac{w}{(1-\alpha)\, z\, (k/l)^{\alpha}} = \mu\left(\tilde{y}\right) MC, \tag{13}$$

where $MC \equiv \frac{w}{(1-\alpha)z(k/l)^{\alpha}}$ is the marginal cost of producing one additional unit of the intermediate good using additional labor and $\mu\left(\tilde{y}\right) = \frac{\sigma\left(\tilde{y}\right)}{\sigma\left(\tilde{y}\right)-1}$ is the markup implied by the demand elasticity in (5). Note that (13) holds regardless of whether the borrowing constraint (9) binds. For an asset-based constraint, the borrowing limit does not depend on labor. For an earnings-based constraint, the borrowing limit depends on labor through earnings, but the Lagrange multiplier on the borrowing constraint rescales marginal revenue and the variable labor cost proportionally, so it cancels from the labor first-order condition. Combining (13) with the demand curve for $p(\tilde{y})$ in (3) and the expression for elasticity $\sigma(\tilde{y})$ in (5) allows us to solve for the intermediate good producer's market share $\tilde{y}$, and thus its markup $\mu(\tilde{y})$, as a function of its marginal cost MC.

We then express the intermediate good producer's marginal cost MC as a decreasing function of its productivity z and an increasing function of the capital wedge τ_k:

$$MC = \frac{(r+\delta)^{\alpha}\, w^{1-\alpha}\, (1+\tau_k)^{\alpha}}{\alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\, z}, \quad \text{where} \quad 1+\tau_k \equiv \frac{\alpha}{1-\alpha}\frac{w}{r+\delta}\frac{l}{k}. \tag{14}$$

The capital wedge τ_k is defined as in Hsieh and Klenow (2009). It is positive if the producer uses "too little" capital relative to labor (so $\frac{l}{k}$ is too high), driving the marginal product of capital too high relative to the cost of capital. In our model, a firm has a positive capital wedge τ_k when its borrowing constraint (9) is binding. In that case, the capital wedge τ_k maps one-to-one to the Lagrange multiplier $\lambda_b \geq 0$ on the borrowing constraint (9): the first-order condition for capital k in the producer's problem (8) implies

$$\tau_k (r+\delta) = \lambda_b \left(1 - \frac{\partial \bar{b}}{\partial k}\right), \tag{15}$$

where $\bar{b} \equiv \max\{\phi^A k, \phi^\pi \pi\}$ is the borrowing limit in (9). As shown in Appendix A, τ_k is strictly increasing in λ_b regardless of which constraint binds: a higher capital wedge corresponds to a larger multiplier on the borrowing constraint. As a result, τ_k is a measure of constraint tightness in the model, and we will use "constraint tightness" to refer to τ_k in the rest of the draft: the higher the capital wedge τ_k is, the more restrictive the borrowing constraint is for the intermediate good producer's capital choices, and hence, the production outcomes.[10] This constraint tightness is determined jointly by the intermediate good producer's net worth and productivity (a, z), as well as the capacity for borrowing against its assets and earnings (ϕ^A, ϕ^π).

Combining the previous steps, we can express differences in intermediate good producers' markups as the differences in their productivity and constraint tightness. The expressions take a particularly simple form if we take an approximation around a symmetric point where all intermediate goods producers have zero capital wedges $\tau_k = 0$, charge a given constant markup μ^*,

[10] We use τ_k rather than λ_b because τ_k connects directly to the Hsieh and Klenow (2009) misallocation formula and can be measured from Census data.

and have the same productivity z. In fact, this approximation generalizes the Hsieh and Klenow (2009) approach based on log-normal distributions and arrives at the exact same formula for TFP losses in Section 5. Under this approximation, we can express differences in intermediate good producers' markups as follows.

Proposition 1. *Consider two intermediate good producers $i, j \in [0,1]$. Under the specification of $\Upsilon(\cdot)$ in (4), to first order, the differences in their markups are given by*

$$\log\mu(i) - \log\mu(j) \approx \frac{\eta}{(\sigma-1)} \underbrace{\left(\log\tilde{y}(i) - \log\tilde{y}(j)\right)}_{\text{differences in market share}} \approx -\frac{\sigma\eta}{\sigma-1+\sigma\eta} \underbrace{\left(\log MC(i) - \log MC(j)\right)}_{\text{differences in marginal cost}} \tag{16}$$

$$\approx \frac{\sigma\eta}{\sigma-1+\sigma\eta} \underbrace{\left(\log z(i) - \log z(j)\right)}_{\text{differences in productivity}} - \frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta} \underbrace{\left(\log(1+\tau_k(i)) - \log\left(1+\tau_k(j)\right)\right)}_{\text{differences in constraint tightness}}. \tag{17}$$

(16) captures the standard properties of the Kimball aggregator. Intermediate good producers with lower marginal costs, and hence higher market shares, face less elastic demand (e.g., in (5)) and have higher markups. (17) shows that markup differences in our model depend on differences in both productivity and constraint tightness. As standard in the literature (e.g., Edmond, Midrigan, and Xu, 2023; Baqaee, Farhi, and Sangani, 2024a,b), higher markups can arise because productive firms have lower marginal costs and larger market shares. Unique to our model, higher markups can also arise because less constrained firms (i.e., firms with lower capital wedges τ_k) have lower marginal costs and higher market shares.

Proposition 1 also illustrates how key parameters impact the dispersion of markups across different intermediate goods producers. A higher η (a higher superelasticity) means that the same differences in market share translate into larger differences in demand elasticity and hence larger differences in markups. In turn, the same differences in productivity and constraint tightness also translate into larger differences in markups. In addition, a higher α (greater importance of capital in the production function (6)) means that constraint tightness impacts markups more: the same differences in constraint tightness also translate into larger differences in markups.

Two key mechanisms behind the negative correlation between markups and constraint tightness Our model features two key mechanisms that explain the negative correlation between markups and constraint tightness, which helps explain the empirical facts in Section 2. First, looser borrowing constraints lower marginal costs (17), allowing firms to attain larger market shares and thus charge higher markups (16). This applies in general for both asset-based and earnings-based borrowing constraints. Second, when firms rely on earnings to borrow, higher

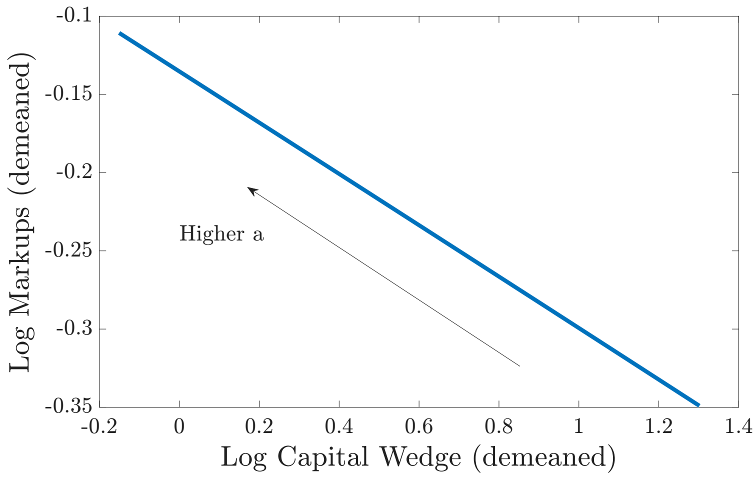
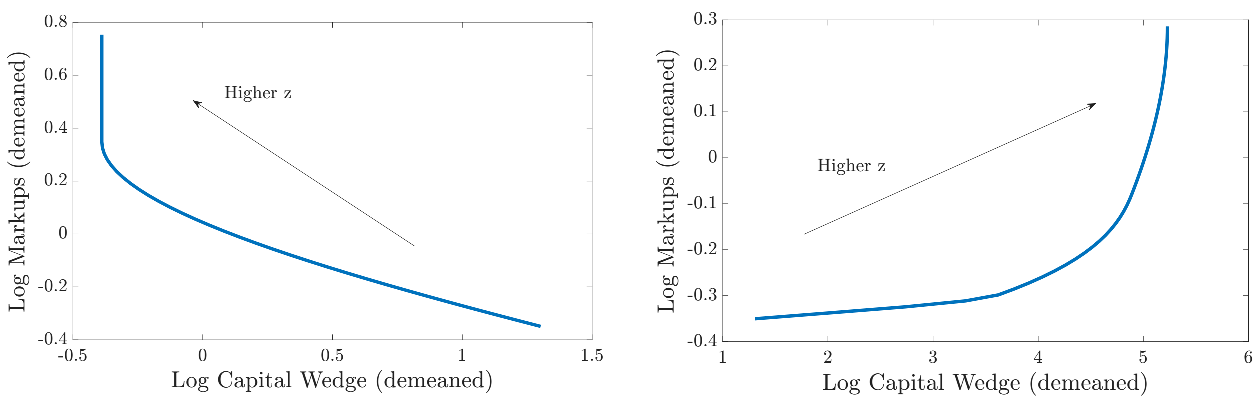


Figure 3: Markups and Constraint Tightness of Firms with Different Net Worth a but the Same Productivity z. This figure plots, for firms with the same productivity ($\log z$ at its mean) but different net worth a, their constraint tightness, measured as log capital wedges $\log(1+\tau_k)$ (demeaned, x-axis), and their log markup $\log\mu$ (demeaned, y-axis). The model parameters are from the baseline calibration explained below.



Panel A. Relevant Earnings-Based Constraints

Panel B. No Earnings-Based Constraints

Figure 4: Markups and Constraint Tightness of Firms with the Same Net Worth a but Different Productivity z. This figure plots, for firms with the same net worth ($a = 0.01$) but differing in their productivity z, their constraint tightness, measured as log capital wedges $\log(1+\tau_k)$ (demeaned, x-axis), and their log markup $\log\mu$ (demeaned, y-axis). The left panel is based on the main calibration explained below, while the right panel reflects the case without earnings-based constraints ($\phi^{\pi} = 0$, holding other parameters fixed).

productivity firms tend to have looser constraints because of their higher earnings. These firms also tend to have higher markups, again through larger market shares and Kimball demand ((16) and (17)). This second mechanism further generates a negative correlation between markups and constraint tightness, but operates only under earnings-based borrowing constraints and is absent under asset-based borrowing constraints.

We now provide simple illustrations of these two mechanisms. The first mechanism is best illustrated by studying firms with the same productivity, z, but different net worth, a. Figure 3 plots their constraint tightness, measured by log capital wedges, $\log(1+\tau_k)$ (demeaned, x-axis), and log markup, $\log \mu$ (demeaned, y-axis).[11] We observe that firms with higher a, which have lower capital wedges and are less constrained, have higher markups.

The second mechanism is best illustrated by studying firms with the same net worth, a, but different productivity, z. Figure 4 plots their constraint tightness, measured in log capital wedges $\log(1+\tau_k)$ (demeaned, x-axis), and log markup $\log \mu$ (demeaned, y-axis). Panel A studies the empirically relevant case where earnings-based borrowing constraints are operative: the capacity for borrowing against earnings, ϕ^{π}, is 3.5, as in our main calibration. We observe that firms with higher z and correspondingly higher markups have lower capital wedges: they are less constrained because they have higher earnings to borrow against. This second mechanism is, however, absent without earnings-based constraints. Panel B plots this case, with $\phi^{\pi} = 0$, holding other parameters fixed. Firms with higher z and correspondingly higher markups have higher capital wedges and are more constrained: more productive firms seek to borrow more, but under asset-based constraints they have the same borrowing capacity as other firms with the same a and lower z.

Both mechanisms explain Fact 1a in Section 2, namely that less constrained firms have higher markups. Because the second mechanism operates only under earnings-based borrowing constraints, the negative relationship between markups and constraint tightness is stronger in industries with low asset liquidation values that rely more on borrowing against earnings, which explains Fact 1b in Section 2. In turn, since higher markup firms in these industries face looser borrowing constraints, they can borrow more and further expand their market shares, raising their markups even higher and increasing overall markup dispersion, explaining Fact 2 in Section 2. In the following, we show that our model not only accounts for these empirical facts qualitatively, but also reproduces them quantitatively.

4.2 Calibration

We calibrate the model in two steps. First, we exogenously assign a subset of parameters using standard values or direct empirical counterparts. Second, we choose the remaining parameters to match moments that discipline firms' borrowing constraint tightness, entrant scale, and superelasticity. We describe each step next.

Assigned parameters. Table 5 lists the parameters that are exogenously assigned. Because a model period is one year, we set the discount factor β to 0.95, following David and Venkateswaran

[11] Figures 3 and 4 are plotted based on the main calibration parameters explained in detail below.

Table 5: Assigned Parameters

Parameter	Description	Source	Value
β	Discount Factor	David and Venkateswaran (2019)	0.95
ϕ^{π}	Capacity for Borrowing against Earnings	Lian and Ma (2021)	3.5
ϕ^{A}	Capacity for Borrowing against Assets	Kermani and Ma (2023a,b)	0.2
δ	Depreciation Rate	David and Venkateswaran (2019)	0.1
r	Interest Rate	Fed Cleveland 10-Year Real Interest Rate	2%
α	Capital Share	David and Venkateswaran (2019)	0.33
ρ	Persistence of Productivity	Sterk, Sedláček, and Pugsley (2021)	0.96
x	Exogenous Exit Rate	Firm Exit Rate Census BDS	9%
σ	Elasticity of Substitution	Broda and Weinstein (2006)	4
w	Wage	Normalize	1

Notes. This table displays parameters that are exogenously assigned in the main calibration.

(2019). We set the capacity for borrowing against assets, ϕ^A, to 0.2, based on the median liquidation value of fixed assets and working capital for Compustat firms, normalized by aggregate book assets, following Kermani and Ma (2023a,b). We set the capacity for borrowing against earnings, ϕ^π, to 3.5, following the contractual evidence in Lian and Ma (2021). Capital depreciates at rate $\delta = 0.1$ annually, following David and Venkateswaran (2019). We set the real interest rate to $r = 2\%$, equal to the historical average of the Cleveland Fed 10-Year real interest rate. We set the capital share to $\alpha = 0.33$, a standard value used, for example, in David and Venkateswaran (2019). We set the persistence of log idiosyncratic productivity to $\rho = 0.96$, following Sterk, Sedláček, and Pugsley (2021). The annual exogenous exit rate is $x = 9\%$, calculated from Census BDS data for all industries in 2023. We set σ, which governs the average demand elasticity of substitution, to 4, following Broda and Weinstein (2006). We normalize the wage w to 1.[12]

Calibrated parameters. In the second step, we choose the parameters listed in Table 6 to match the moments in Table 7. While all calibrated parameters are jointly chosen to match the full set of targets, it is useful to organize identification around three groups of moments.

First, to discipline how close firms are to their borrowing constraints in the stationary distribution, we target the ratio of aggregate earnings to aggregate debt ($\mathbb{E}[\pi]/\mathbb{E}[b]$), calculated using Quarterly Financial Report (QFR) firms; the median firm's earnings-to-debt ratio (median π/b), calculated using Compustat firms; and the median firm's debt-to-assets ratio (median b/k), also calculated using Compustat firms.[13] These moments are primarily informative about the

[12] We verified that the normalization does not affect our results.

[13] We use QFR to calculate aggregate earnings relative to aggregate debt, as it covers the full population of firms,

Table 6: Calibrated Parameters

Parameter	Description	Value
c	Fixed Operating Cost	0.46
D	Industry Demand	1.21
σ_e^2	Var. of Idiosyncratic Productivity Shock	0.10
a^{ent}	Entrants' Initial Net Worth	0.10
ψ_d	Equity Issuance Cost	0.01
η	Governs Superelasticity	0.73

Notes. This table displays parameters that are chosen to match moments in Table 7.

fixed operating cost c, industry demand $D = PY$, and the variance of the idiosyncratic productivity shock σ_e^2. Intuitively, the aggregate earnings-to-debt ratio is informative about industry demand $D = PY$, which raises the total amount of industry earnings. The median earnings-to-debt ratio is informative about the variance of the idiosyncratic productivity shock σ_e^2, which drives the gap between the median firm's earnings-to-debt ratio and the ratio of aggregate earnings to aggregate debt. The median debt-to-asset ratio in the data is 0.21, close to ϕ^A, the capacity for borrowing against assets. To fit this, the model requires a sufficiently high fixed operating cost c so that the median firm's borrowing capacity against assets exceeds its capacity to borrow against earnings.[14]

The second set of moments governs entrants: the entrant employment share $(x \cdot l^{ent}/\mathbb{E}[l])$,[15] calculated based on the share of employment in firms less than one year old using the Census Business Dynamics Statistics database for all sectors in 2023 (2%); and the entrant's capital relative to the average firm's capital $(k^{ent}/\mathbb{E}[k])$, taken directly from Ottonello and Winberry (2024) (4%). These moments are informative about entrants' initial net worth a^{ent} and the cost of equity issuance ψ_d. A higher value of a^{ent} raises the entrant employment share. Entrants face binding borrowing constraints, and costly equity issuance becomes the relevant margin that governs how much capital they can acquire beyond their initial net worth. In the model, a lower cost of equity issuance means higher entrant capital relative to the average firm's capital $(k^{ent}/\mathbb{E}[k])$.

including private ones. However, as QFR does not publish firm-level data, we use Compustat, which covers public firms only, to calculate the median firm's debt-to-assets ratio and earnings-to-debt ratio.

[14] Under Kimball demand, there exists a choke price at and above which a firm's market share implied by the demand curve (3) is zero (Dotsey and King, 2005; Klenow and Willis, 2016). In the calibrated model, firms with sufficiently low productivity do not produce because they cannot profitably price below the choke price. Because markups are only defined for firms that produce, we follow the standard practice and compute model moments using the distribution of producing firms (Edmond, Midrigan, and Xu, 2023).

[15] The entry rate is equal to the exit rate x.

Table 7: Calibration Targets

Calibration Target	Source	Data	Model
Agg Earnings / Debt ($\mathbb{E}[\pi]/\mathbb{E}[b]$)	QFR	0.38	0.38
Median Earnings/Debt (π/b)	Compustat	0.26	0.26
Median Debt/Assets (b/k)	Compustat	0.21	0.20
Entrant Emp Share ($x \cdot l^{ent}/\mathbb{E}[l]$)	Census BDS	2%	2%
Entrant Relative Capital ($k^{ent}/\mathbb{E}[k]$)	Ottonello and Winberry (2024)	4%	5%
Markup Elasticity $\left(\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\Big\|_{\tilde{y}=1}\right)$	Beck and Lein (2020)	-0.99	-0.97

Notes. This table displays the empirical moments targeted in our calibration and the corresponding values generated by the model.

Third, we target the markup elasticity defined as $\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\Big|_{\tilde{y}=1}$, following Beck and Lein (2020). In our model, this elasticity equals $-\frac{\sigma\eta}{\sigma-1}$ and is therefore informative about η, which governs superelasticity through (5). The empirical estimate is -0.99. Because this estimate is imprecisely estimated (the 25th to 75th percentile range in Beck and Lein (2020) is $[-2.14, 0.11]$) and somewhat controversial in the literature, we also assess robustness to alternative values of η below, including one that matches the superelasticity in Edmond, Midrigan, and Xu (2023).

Table 7 shows that our model matches the targeted moments well. The calibrated parameters in Table 6 are comparable to existing estimates in the literature. For example, our calibrated variance of the idiosyncratic shock is $\sigma_e^2 = 0.10$ whereas David and Venkateswaran (2019) calibrates a value of 0.08. Table IA9 compares empirical moments not included in the calibration targets with the corresponding values generated by our model. The model matches the persistence of log markups reasonably well (0.79 vs. 0.90 in the data) and approximates the variance in log net worth (3.67 vs. 4.34 in the data). It also generates plausible shares of employment, sales, and assets accounted for by firms in the bottom decile.[16]

4.3 Quantitative Fit

With the two key mechanisms behind the negative correlation between markups and constraint tightness described in Subsection 4.1, our model can quantitatively explain Fact 1a, Fact 1b, and Fact 2 in Section 2, even though it does not directly target any of these facts.

[16] We use data moments on the share of bottom decile firms for assets and sales from the Internal Revenue Service Statistics of Income (SOI) publications, and that for employment from Census Business Dynamics Statistics (BDS), like Kwon, Ma, and Zimmermann (2024). These datasets cover the population of U.S. firms. Results are very similar in Compustat data too.

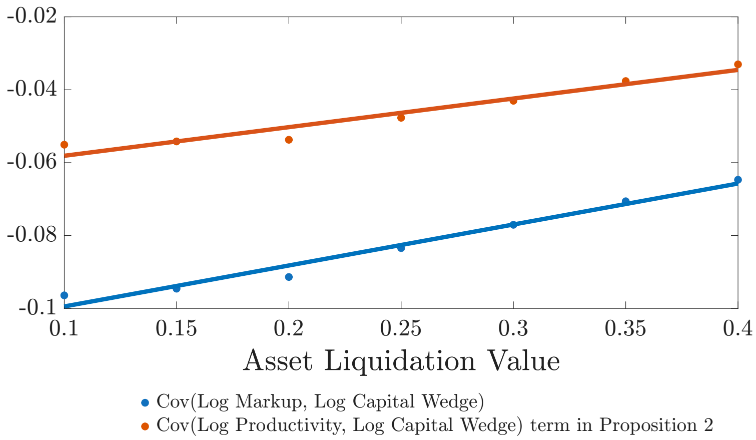


Figure 5: Less Constrained Firms Have Higher Markups, Especially When Firms Rely on Earnings to Borrow. The blue line plots $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ in the model (y-axis) against ϕ^A (x-axis). The red line plots $\frac{\sigma\eta}{\sigma-1+\sigma\eta}Cov\left(\log z, \log\left(1+\tau_k\right)\right)$ in Proposition 2 in the model (y-axis) against ϕ^A (x-axis). When we vary ϕ^A, we recalibrate β while keeping all other parameters fixed, so that aggregate saving $\mathbb{E}[a]$ remains fixed.

Fact 1a and Fact 1b First, our model explains Fact 1a, which shows that less constrained firms have higher markups. In the benchmark calibration with $\phi^A = 0.2$, we find that firms' markups and their constraint tightness (measured using the capital wedge) are negatively correlated: $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right) \approx -0.09$.

Second, our model also explains Fact 1b, which shows that the negative relationship between constraint tightness and markups is stronger in industries with low asset liquidation values (corresponding to a low capacity for borrowing against assets, ϕ^A, in the model), where firms rely more on borrowing against earnings.

To apply our model to explain this fact, we solve for the industry equilibria for different values of ϕ^A. When we vary ϕ^A, we recalibrate β such that aggregate saving $\mathbb{E}[a]$ remains fixed for different values of ϕ^A. In the data, industry-level leverage (e.g., total debt in the industry relative to total assets or total earnings) does not correlate significantly with the industry asset liquidation value ϕ^A (Table IA7); this is consistent with the empirical interpretation of ϕ^A in Section 2, which affects the extent of reliance on earnings to borrow, rather than systematic variations in overall borrowing or constraint tightness. Without recalibration, however, industry-level leverage in the model would vary significantly with ϕ^A, so that changes in ϕ^A would also alter industry-level borrowing constraint tightness, which is inconsistent with the data. The recalibration thus ensures that when we vary ϕ^A, we vary the extent of reliance on earnings to borrow, rather than generating systematic variations in overall borrowing and constraint tightness.

We keep other parameters (particularly the capacity for borrowing against earnings, ϕ^π)

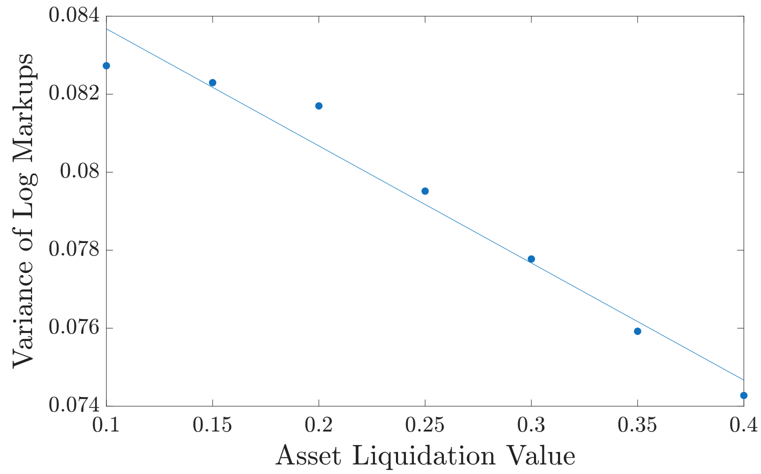


Figure 6: Higher Markup Dispersion When Firms Rely More on Earnings to Borrow. The figure plots $Var(\log\mu)$ in the model (y-axis) against ϕ^A (x-axis). When we vary ϕ^A, we recalibrate β while keeping all other parameters fixed, so that aggregate saving $\mathbb{E}[a]$ remains fixed.

fixed. This approach is motivated by the evidence in Lian and Ma (2021) and Kermani and Ma (2023a) that ϕ^A differs significantly across industries due to cross-industry variations in asset specificity, whereas ϕ^π remains roughly constant across industries.

In Figure 5, we plot how the covariance between log markup and constraint tightness (measured using log capital wedge), $Cov(\log\mu, \log(1+\tau_k))$, in the model (y-axis) varies with asset liquidation value ϕ^A (x-axis), shown by the blue line. We find that the negative relationship between firms' markups and constraint tightness is stronger when ϕ^A is lower, so firms rely more on earnings to borrow. This pattern arises because the second mechanism behind the negative correlation between markups and constraint tightness is stronger when ϕ^A is lower. The slope of $Cov(\log\mu, \log(1+\tau_k))$ with respect to ϕ^A in Figure 5 is 0.11. This magnitude matches the empirical counterpart well: in Column (1) of Panel B of Table 3, the regression coefficient of $Cov(\log\mu, \log(1+\tau_k))$ in an industry (measured using the covariance of log markup with log production worker hours per unit of capital) on the industry asset liquidation value in Census data is around 0.10.[17]

Fact 2 Finally, our model explains Fact 2 in Section 2, which shows that markup dispersion is higher in industries with lower asset liquidation value ϕ^A and greater reliance on earnings to borrow. Figure 6 plots the variance of log markup, $Var(\log\mu)$, in the model (y-axis) against ϕ^A (x-axis), for the industry equilibrium solved above under different values of ϕ^A. Consistent with Fact 2, we indeed find that markup dispersion is higher when ϕ^A is lower and firms rely more

[17] Note that we cannot directly compare $Cov(\log\mu, \log(1+\tau_k))$ in the model with the covariance between log markup and the cash/assets ratio studied in Table 1, as the cash/assets ratio does not directly map to $\log(1+\tau_k)$ in the model.

on earnings to borrow. The slope of $Var(\log\mu)$ with respect to ϕ^A in Figure 6 is −0.03.

The slope of $Var(\log\mu)$ with respect to ϕ^A in the model (around −0.03) is smaller than its empirical counterpart in the Census data in Table 4 (around −0.09).[18] This smaller magnitude in the model could arise from the well-known fact that markup dispersion in models is often smaller than its empirical counterpart, due to measurement errors of markups in the data which would increase the empirical dispersion (Bils, Klenow, and Ruane, 2021). We hence assess how markup measurement errors affect measures of markup dispersion in the data and construct a version of markup dispersion, free from measurement errors, that is comparable to its model counterpart. Specifically, following the methodology in Bloom and Van Reenen (2007) and Aghion et al. (2025), we use the fact that we have (log) markup measures for the same firm using two different data sources: Census $\log\mu^{Census}$, and Compustat $\log\mu^{Compustat}$ (for Compustat firms that can be matched to the Census data). If the measurement errors in these two datasets are independent, namely,

$$\log\mu^{Census} = \log\mu + \epsilon^{Census} \quad \text{and} \quad \log\mu^{Compustat} = \log\mu + \epsilon^{Compustat},$$

and if ϵ^{Census} and $\epsilon^{Compustat}$ are independent of the true markup $\log(\mu)$ and of each other, we can recover the true markup dispersion by first regressing $\log\mu^{Compustat}$ on $\log\mu^{Census}$ and obtaining the coefficient

$$\xi = \frac{Cov\left(\log\mu^{Census}, \log\mu^{Compustat}\right)}{Var\left(\log\mu^{Census}\right)} = \frac{Var(\log\mu)}{Var\left(\log\mu^{Census}\right)}.$$

In this case, the true markup dispersion is

$$Var(\log\mu) = \xi \cdot Var\left(\log\mu^{Census}\right).$$

Empirically, we merge the firm level markup measures from Census and Compustat, and find that ξ is around 0.4, as shown in Table IA6.

Furthermore, Table IA8 checks that ξ does not vary significantly with the industry asset liquidation value ϕ^A, which is on the x-axis of Figures 2 and IA3 (as well as the model counterpart Figure 6). Table IA8 also checks that ξ does not vary significantly with variances of log markup (measured using either Compustat data or Census data), which are on the y-axis of those figures. In other words, the key relationships in Facts 1b and 2 should not be driven by cross-industry variation in the share of measured markup dispersion attributable to measurement error.

[18]We compare the slope of markup dispersion with respect to ϕ^A to its Census counterpart in Table 4, rather than to results using Compustat in Table 2, since the Census data provide a better markup measure and broader coverage including both public and private firms.

Accordingly, the slope of how much the true markup dispersion $Var(\log\mu)$ varies with ϕ^A is given by ξ times the slope of how much the measured markup dispersion $Var(\log\mu^{Census})$ varies with ϕ^A, which would be around $-0.036 = 0.4 \times -0.09$, based on estimates for ξ in Table IA6 and the coefficient in Table 4. This is very close to our model counterpart, -0.03, in Figure 6.

More broadly, measurement error concerns extend beyond markup dispersion. The variance of capital wedges (and possibly also the covariance between markups and capital wedges) is also likely contaminated by measurement error in the data (David and Venkateswaran, 2019). For this reason, we do not directly target these moments in our calibration in Section 4.2.

4.4 Unpacking the Quantitative Fit

The role of the two key mechanisms The following decomposition further clarifies how the two key mechanisms in Section 4.1 behind the negative correlation between markups and constraint tightness help the model explain Fact 1b and Fact 2 quantitatively. We start with Fact 1b, which finds that the covariance between log markup and constraint tightness (measured in log capital wedge), $Cov(\log\mu, \log(1+\tau_k))$, becomes more negative when the asset liquidation value ϕ^A is lower.

Proposition 2. *Under the specification of* $\Upsilon(\cdot)$ *in* (4)*, to second order, the covariance between log markup and constraint tightness (measured in log capital wedge) is given by*

$$Cov(\log\mu, \log(1+\tau_k)) \approx -\frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta} Var(\log(1+\tau_k)) + \frac{\sigma\eta}{\sigma-1+\sigma\eta} Cov(\log z, \log(1+\tau_k)). \tag{18}$$

This decomposition follows from (17) in Proposition 1. The first term in (18) corresponds to the first key mechanism: less constrained firms have lower marginal costs and hence higher markups. As a result, a greater dispersion in constraint tightness (higher $Var(\log(1+\tau_k))$) leads to a more negative relationship between markups and constraint tightness (more negative $Cov(\log\mu, \log(1+\tau_k))$). The second term in (18) corresponds to the second key mechanism: firms with higher productivity have higher markups. As a result, a more negative relationship between productivity and constraint tightness (more negative $Cov(\log z, \log(1+\tau_k))$) leads to a more negative relationship between markups and constraint tightness (more negative $Cov(\log\mu, \log(1+\tau_k))$).

How the term $Cov(\log z, \log(1+\tau_k))$ in (18) varies with ϕ^A reveals how ϕ^A influences the strength of the second key mechanism. When ϕ^A is lower and firms rely more on earnings to borrow, the second mechanism is stronger: $Cov\left(\log z, \log\left(1+\tau_k\right)\right)$ becomes more negative because firms with higher productivity have higher earnings and are less constrained, as illustrated in Panel A of Figure 4. When ϕ^A is higher and firms rely less on earnings to borrow, the second mechanism is weaker: $Cov\left(\log z, \log\left(1+\tau_k\right)\right)$ becomes less negative (and can even turn positive) because firms with higher productivity seek to borrow more and therefore may be more constrained, similar to Panel B of Figure 4.

The red line in Figure 5 shows how the $Cov\left(\log z, \log\left(1+\tau_k\right)\right)$ term in (18) varies with ϕ^A. Its slope with respect to ϕ^A is 0.08, which accounts for most of the 0.11 slope of $Cov\left(\log \mu, \log\left(1+\tau_k\right)\right)$ with respect to ϕ^A, while the slope of the first term $-\frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta} Var\left(\log\left(1+\tau_k\right)\right)$ with respect to ϕ^A rounds to 0.04. Thus, the slope of $Cov\left(\log \mu, \log\left(1+\tau_k\right)\right)$ with respect to ϕ^A is largely driven by how ϕ^A modulates the strength of the second key mechanism.

We have also verified that the approximation formula (18) in Proposition 2 is accurate: the slope of the exact $Cov\left(\log \mu, \log\left(1+\tau_k\right)\right)$ with respect to ϕ^A is 0.11, whereas the slope of the approximate $Cov\left(\log \mu, \log\left(1+\tau_k\right)\right)$ (constructed by adding the two terms in (18)) with respect to ϕ^A also rounds to 0.11.

We now turn to the role of the two key mechanisms in explaining Fact 2, which shows that markup dispersion is higher when ϕ^A is lower and firms rely more on earnings to borrow. To build intuition and connect with the discussion above around Proposition 2, we provide decompositions linking the variance of log markup with (i) the covariance between log productivity and constraint tightness, which determines the strength of the second mechanism and (ii) the covariance between log markup and constraint tightness (measured by the log capital wedge), the central object in Fact 1b and Proposition 2.

Proposition 3. *Under the specification of $\Upsilon(\cdot)$ in (4), to second order, the variance of log markup is given by*

$$Var\left(\log \mu\right) \approx \left[\frac{\sigma\eta}{\sigma-1+\sigma\eta}\right]^2 \left[Var\left(\log z\right) + \alpha^2 Var\left(\log\left(1+\tau_k\right)\right) - 2\alpha Cov\left(\log z, \log\left(1+\tau_k\right)\right)\right] \tag{19}$$

$$\approx \left[\frac{\sigma\eta}{\sigma-1+\sigma\eta}\right]^2 Var\left(\log z\right) - \left[\frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta}\right]^2 Var\left(\log\left(1+\tau_k\right)\right) \underbrace{-\frac{2\alpha\sigma\eta}{\sigma-1+\sigma\eta} Cov\left(\log \mu, \log\left(1+\tau_k\right)\right)}_{>0 \text{ if higher markup firms less constrained}}. \tag{20}$$

(19) follows from how intermediate good producers' productivity and constraint tightness impact their markups in (17). It links $Var\left(\log \mu\right)$ with the $Cov\left(\log z, \log\left(1+\tau_k\right)\right)$ term, which captures the strength of the second key mechanism behind the negative correlation between

markups and constraint tightness as explained after Proposition 2. The slope of the $Cov(\log z, \log(1+\tau_k))$ term in (19) with respect to ϕ^A is -0.03 and accounts for the -0.03 slope of $Var(\log\mu)$ with respect to ϕ^A. Similar to Figure 5, how ϕ^A impacts the strength of the second key mechanism is the key to explaining the pattern in Figure 6.

(20) uses (18) to replace $Cov(\log z, \log(1+\tau_k))$ and directly links $Var(\log\mu)$ with $Cov(\log\mu, \log(1+\tau_k))$, the central object of Fact 1b and Figure 5. It shows that $Var(\log\mu)$ decreases with $Cov(\log\mu, \log(1+\tau_k))$. Hence, from Fact 1b and Figure 5, where a lower ϕ^A (more reliance on earnings to borrow) implies a more negative $Cov(\log\mu, \log(1+\tau_k))$, a lower ϕ^A also implies a higher markup dispersion $Var(\log\mu)$. Intuitively, if larger firms with higher markups are also less constrained, they produce even more through borrowing and hence achieve even higher market share and hence markups. This leads to greater markup dispersion. Because $Cov(\log\mu, \log(1+\tau_k))$ is more negative in industries with lower ϕ^A and greater reliance on earnings to borrow, as shown in Figure 5, markup dispersion is also higher in those industries.

We have also verified that the approximation in Proposition 3 is accurate. The slope of the exact $Var(\log\mu)$ with respect to ϕ^A is -0.03. The slope of the approximate $Var(\log\mu)$, constructed by adding the three terms in (20), with respect to ϕ^A is also -0.03.

Key features of the calibration We now investigate the key features of the calibration that help explain the empirical Facts 1a, 1b, and 2 regarding the connection between constraint tightness and markups in Section 2.

First, we need a sufficiently high η (and hence sufficiently high superelasticity) to ensure that markup varies sufficiently with market share and, consequently, constraint tightness. This generates sufficient markup variation across firms, which enables our two key mechanisms behind the negative correlation between markups and constraint tightness to operate, and is crucial to explain all of Fact 1a, Fact 1b, and Fact 2. In Table 8, we examine how the key results in Figures 5 and 6 vary across different values of η, corresponding to various percentiles of Beck and Lein (2020)'s estimates. Our baseline calibration sets $\eta = 0.73$ so that the implied markup elasticity $\left.\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\right|_{\tilde{y}=1}$ is close to the mean reported by Beck and Lein (2020). Table 8 shows results for lower values of η corresponding to the median ($\eta = 0.47$) and 75th-percentile markup elasticities ($\eta = 0$, imposing nonnegative η) from Beck and Lein (2020). It also shows the results for $\eta = 0.25$, which is set to match the superelasticity of 0.16 used in Edmond, Midrigan, and Xu (2023).[19] When we vary η, we aim to isolate the impact of changing superelasticities without

[19] Edmond, Midrigan, and Xu (2023) features the Klenow-Willis demand function with parameters σ^{KW} and ϵ^{KW}. They set the superelasticity $\epsilon^{KW}/\sigma^{KW}$ to 0.16 to match an empirical relationship between markups and sales share

Table 8: Alternative Values of Parameter Governing Superelasticity η

	Baseline	Lower		Zero
	$\eta = 0.73$	$\eta = 0.47$	$\eta = 0.25$	$\eta = 0$
Slope of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ w.r.t. ϕ^A	0.11	0.10	0.07	0
Slope of the $Cov\left(\log z, \log(1+\tau_k)\right)$ Term in (18) w.r.t. ϕ^A	0.08	0.09	0.06	0
Slope of $Var\left(\log\mu\right)$ w.r.t. ϕ^A	-0.03	-0.02	-0.01	0
Slope of the $Cov\left(\log z, \log(1+\tau_k)\right)$ Term in (19) w.r.t. ϕ^A	-0.03	-0.02	-0.01	0

Notes. The table displays results for alternative values of the parameter η. Specifically, $\eta = 0.47$ and $\eta = 0$ correspond to the median and 75th percentile (imposing nonnegative η) markup elasticities $\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\Big|_{\tilde{y}=1}$ reported in Beck and Lein (2020). $\eta = 0.25$ is set to match the value of superelasticity used in Edmond, Midrigan, and Xu (2023). When we vary η, we recalibrate σ while keeping all other parameters fixed, so that aggregate markups remain at their baseline level.

inducing systematic variations in average elasticities or markups. Accordingly, we recalibrate σ, which governs average demand elasticities and therefore average markups, while keeping all other parameters fixed, so that aggregate markups remain fixed at their baseline level.

Table 8 shows that a lower superelasticity ($\eta = 0.47$ or $\eta = 0.25$) reduces the magnitudes of the slopes of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ with respect to ϕ^A. This is natural because a lower η moderately reduces markup differences across firms (see Proposition 1), thereby lowering markup dispersion. But this does not fundamentally change the share of the slopes attributable to the slopes of the respective $Cov\left(\log z, \log(1+\tau_k)\right)$ terms in (18) and (19) with respect to ϕ^A, which govern how ϕ^A affects the strength of the second key mechanism, as explained after Propositions 2 and 3. In other words, a lower η does not alter the qualitative properties of Figures 5 and 6 or the economics underlying the two key mechanisms behind the negative correlation between markups and constraint tightness. In the limit case when $\eta = 0$, markup dispersion across firms disappears, so naturally the slopes of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ with respect to ϕ^A are zero.

Second, firms need to have an empirically relevant capacity for borrowing against earnings ($\phi^\pi = 3.5$) to explain the cross-industry variations in Fact 1b and Fact 2. As discussed above, the way $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ vary with ϕ^A hinges on how ϕ^A affects firms' reliance on earnings for borrowing: when ϕ^A is low, firms rely more on earnings, which strengthens the second key mechanism behind the negative correlation between markups and

and solve the model for various values of σ^{KW}. To a first-order approximation, the Klenow-Willis demand function with parameters σ^{KW} and ϵ^{KW} is the same as the Dotsey-King demand function with parameters $\sigma = \sigma^{KW}$ and $\eta = \epsilon^{KW}/(\sigma^{KW}-1)$. To facilitate comparison with the baseline calibration, we set $\sigma = 2.8$ and $\eta = 0.25$ so that the aggregate markup stays the same as in the baseline and the superelasticity $\eta\frac{\sigma-1}{\sigma}$ equals 0.16 in Edmond, Midrigan, and Xu (2023).

Table 9: Alternative Values of Capacity for Borrowing against Earnings ϕ^{π}

	No EBC $\phi^{\pi}=0$	Baseline $\phi^{\pi}=3.5$	Higher EBC Capacity $\phi^{\pi}=5$
Slope of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ w.r.t. ϕ^A	0.01	0.11	0.05
Slope of $Var\left(\log\mu\right)$ w.r.t. ϕ^A	-0.00	-0.03	-0.01

Notes. The table displays results for alternative values of the capacity for borrowing against earnings ϕ^{π}.

constraint tightness; when ϕ^A is high, this mechanism is weaker. Table 9 explores how results in Figures 5 and 6 change with different values of ϕ^{π}. Without the option of borrowing against earnings ($\phi^{\pi}=0$), ϕ^A no longer alters firms' reliance on earnings, so varying ϕ^A leads to significantly smaller changes in both $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$. Similarly, when the capacity for borrowing against earnings is counterfactually high ($\phi^{\pi}=5$), firms rely solely on earnings regardless of ϕ^A, so varying ϕ^A again produces smaller changes in $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$.

Third, a sufficiently large variance of the idiosyncratic productivity shock, σ_e^2, is also important for the model to quantitatively explain empirical Facts 1a, 1b, and 2. A higher σ_e^2 leads to greater heterogeneity across firms in the stationary distribution $G(a,z)$, leading to more markup dispersion and a greater scope for the two key mechanisms behind the negative correlation between markups and constraint tightness. In our baseline calibration, the value $\sigma_e^2=0.1$ is close to that in the literature, e.g., David and Venkateswaran (2019). In Table IA10, we present an alternative calibration where we double the variance of the idiosyncratic productivity shock to $\sigma_e^2=0.2$, while keeping all other parameters fixed. This value is also closer to $\sigma_e^2=0.25$ in Edmond, Midrigan, and Xu (2023). Table IA10 shows that a larger σ_e^2 leads to even steeper slopes of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ with respect to ϕ^A in the model: the coefficients increase substantially under this alternative calibration.

5 Allocative Efficiency

5.1 The Planner's Problem

We now study our model's implications for allocative efficiency. The planner allocates the industry's total labor and capital, L and K, across intermediate good producers to maximize industry final output, subject to the same technology of the final and intermediate good producers, as given in (2) and (6), but not the borrowing constraints.

$$\max_{\{l^P(i),k^P(i)\}_{i\in[0,1]}} Y^P \quad s.t. \quad \int \Upsilon\left(\frac{y^P(i)}{Y^P}\right) di = 1, \quad y^P(i) = z(i)\left(k^P(i)\right)^{\alpha}\left(l^P(i)\right)^{1-\alpha}, \tag{21}$$
$$L = \int l^P(i)\, di, \quad K = \int k^P(i)\, di.$$

As we focus on allocative efficiency, the planner takes the industry's total labor and capital as given, $L = \int l(i)\, di$ and $K = \int k(i)\, di$, from the industry equilibrium studied in Sections 3 and 4.

Consider two intermediate good producers, $i, j \in [0,1]$. Under the specification of $\Upsilon(\cdot)$ based on Dotsey and King (2005) in (4), the differences in their market shares in the planner's problem, $\tilde{y}^P(i) = y^P(i)/Y^P$ and $\tilde{y}^P(j) = y^P(j)/Y^P$, are then linked by their relative productivity,

$$\log\left(\tilde{y}^P(i) + \frac{\eta}{1-\eta}\right) - \log\left(\tilde{y}^P(j) + \frac{\eta}{1-\eta}\right) = \sigma(1-\eta)\left(\log(z(i)) - \log(z(j))\right). \tag{22}$$

This differs from the equilibrium differences in their relative market shares in two ways, as the latter also depend on their relative markups and relative constraint tightness.

$$\begin{aligned}\log\left(\tilde{y}(i) + \frac{\eta}{1-\eta}\right) - \log\left(\tilde{y}(j) + \frac{\eta}{1-\eta}\right) &= \sigma(1-\eta)\left(\log z(i) - \log z(j)\right) \\ &\quad - \sigma(1-\eta)\left(\log\mu(i) - \log\mu(j) + \alpha\left(\log(1+\tau_k(i)) - \log(1+\tau_k(j))\right)\right).\end{aligned} \tag{23}$$

The following proposition summarizes the difference between the intermediate good producer's equilibrium market share and its market share in the planner's problem.

Proposition 4. *Consider the intermediate good producer $i \in [0,1]$. Under the specification of $\Upsilon(\cdot)$ in (4), to first order, the difference between its equilibrium market share and market share in the planner's problem is given by*

$$\log\tilde{y}(i) - \log\tilde{y}^P(i) \approx -\sigma\underbrace{\left(\log\mu(i) - \int \log\mu(j)\, dj\right)}_{\textit{markup relative to avg}} - \sigma\alpha\underbrace{\left(\log(1+\tau_k(i)) - \int \log(1+\tau_k(j))\, dj\right)}_{\textit{constraint tightness relative to avg}}. \tag{24}$$

As is standard in the literature (e.g., Edmond, Midrigan, and Xu, 2023; Baqaee, Farhi, and Sangani, 2024a,b), the first term in (24) captures the force that firms with above-average markups are too small ($\tilde{y}(i) < \tilde{y}^P(i)$). However, as emphasized in our paper, these firms with higher markups are less constrained. As a result, the second term in (24), stemming from constraint tightness, moves these higher-markup firms' market share closer to its planner counterpart.[20]

[20]There is a possibility that the second term in (24) is so strong that it dominates, making higher-markup firms too large relative to the planner. For firms that differ in productivity z but share the same net worth a, high markup firms are those with high productivity and remain too small (the first term in (24) dominates). Conversely, when we consider firms with the same productivity z but different net worth a, high markup firms face looser constraints and are instead too large (the second term in (24) dominates). Given our calibration in Section 4.2 and averaging across all firms, overall higher-markup firms still appear too small, $Cov\left(\log\tilde{y} - \log\tilde{y}^P, \log\mu\right) < 0$; thus the first term continues to dominate the second, yet the second term is strong enough to significantly reduce the TFP loss from markup dispersion, as shown below.

This force has important allocative efficiency implications: it significantly lowers the TFP loss from markup dispersion.

5.2 TFP Losses from Markup Dispersion

To illustrate this, we calculate equilibrium TFP relative to the planner's TFP. For this purpose, we define the equilibrium TFP and its counterpart in the planner's problem as

$$Z = \frac{Y}{K^{\alpha}L^{1-\alpha}} \quad \text{and} \quad Z^P = \frac{Y^P}{K^{\alpha}L^{1-\alpha}}. \tag{25}$$

The (value-added) TFP loss $\log Z^P - \log Z$ in our model serves as a measure of allocative efficiency: it captures the misallocation of labor and capital across intermediate good producers generated by the dispersion in markups and borrowing constraints, while leaving aside distortions stemming from the aggregate markup level that influences the industry's total demand for capital and labor. This is consistent with Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Peters (2020), and Aghion et al. (2023).

We now express this TFP loss as a function of the covariance between the log markup and constraint tightness (measured in the log capital wedge) and their variances, the key objects we studied in Section 4.

Proposition 5. *Under the specification of* $\Upsilon(\cdot)$ *in* (4)*, to second order, the TFP loss is*

$$\underbrace{\log Z^P - \log Z}_{\text{Total TFP loss}} \approx \underbrace{\frac{\sigma}{2} Var\left(\log \mu\right)}_{\text{markup dispersion}} + \underbrace{\frac{\sigma\alpha^2 + \alpha(1-\alpha)}{2} Var\left(\log(1+\tau_k)\right)}_{\text{dispersion in constraint tightness}} \tag{26}$$

$$+ \underbrace{\sigma\alpha Cov\left(\log\mu, \log(1+\tau_k)\right)}_{<0 \text{ if higher-markup firms less constrained}}$$

The TFP loss formula (26) is essentially the same formula for the TFP loss in Hsieh and Klenow (2009), re-expressed for our purposes. Compared to Hsieh and Klenow (2009), we extend in two dimensions. First, we generalize the formula beyond the CES demand case in the original derivation.[21] Second, we consider a second-order approximation, so we do not need the underlying variables to be log-normally distributed.

(26) illustrates how introducing borrowing constraints mitigates TFP losses from markup dispersion. Without borrowing constraints, markup dispersion causes substantial TFP losses: the first term in (26) is positive because high markup firms are too small (the first term in (24)).

[21] The proof focuses on the (4) specification of $\Upsilon(\cdot)$, but we have also verified that the same formula applies to the case of general Kimball demand.

With borrowing constraints, high markup firms also face looser constraints (Fact 1a) as a consequence of the two key mechanisms behind the negative correlation between markups and constraint tightness. These forces help high markup firms borrow and produce more, moving their market shares closer to the planner's allocation. This mitigates TFP losses from markup dispersion: the third term in (26) is negative.

In the benchmark calibration where the capacity for borrowing against assets is set at $\phi^A = 0.2$, we can decompose the TFP loss in (26) as:

$$\underbrace{12.0\%}_{\text{TFP loss}} \approx \underbrace{16.3\%}_{\text{markup dispersion}} + \underbrace{7.8\%}_{\text{dispersion in constraint tightness}} - \underbrace{12.1\%}_{\text{higher-markup firms less constrained}}. \tag{27}$$

In other words, the net TFP loss from markup dispersion, defined as the sum of the first and third terms in (26), is just 16.3% – 12.1% = 4.3%. The fact that high markup firms are less constrained (the third term in (26)) mitigates the TFP loss from markup dispersion by 12.1%/16.3% ≈ 73.8%. Equation (27) also provides an accurate approximation of the TFP loss: the three terms on the right-hand side sum to 12.0%, close to the exact loss of 9.7% calculated from (25). We evaluate all three terms in (26) using model-implied moments rather than directly substituting their empirical counterparts, as the empirical counterparts of these three terms are likely to be subject to measurement error (Bils, Klenow, and Ruane, 2021; David and Venkateswaran, 2019).

Although our focus is the latter fraction, which illustrates how much the two key mechanisms behind the negative correlation between markups and constraint tightness can mitigate the TFP loss from markup dispersion, the level of value-added TFP loss from markup dispersion is within the range estimated in the literature. In the above decomposition, TFP loss from markup dispersion is 16.3% before and 4.3% after netting out the covariance term. These numbers are smaller than the 20% loss in Baqaee and Farhi (2020) because they use the empirical markup dispersion, which is higher due to measurement errors. Edmond, Midrigan, and Xu (2023) focuses on model-implied markup dispersion and finds a TFP loss of 9% when the cost-weighted aggregate markup is 1.45, a level estimated by De Ridder, Grassi, and Morzenti (2026) using price data from France.

Furthermore, the cross-industry differences documented in Fact 1b and Fact 2 (studied in Figures 5-6) also affect the extent to which the borrowing constraint channel mitigates TFP losses from markup dispersion. In Table 10, we study this question under different values of ϕ^A, based on the industry equilibria under different values of ϕ^A behind Figures 5 and 6.

When the capacity for borrowing against assets (ϕ^A) is lower, firms rely more on earnings to borrow: markup dispersion, $Var(\log \mu)$, is higher (Fact 2); consequently, the gross TFP loss

Table 10: TFP Losses from Markup Dispersion and Mitigation for Different Values of ϕ^A

	$\phi^A = 0.1$	$\phi^A = 0.2$	$\phi^A = 0.3$	$\phi^A = 0.4$
Total TFP Loss in (26)	12.3%	12.0%	12.4%	12.8%
Gross TFP loss from Markup Dispersion in (26)	16.5%	16.3%	15.6%	14.9%
Covariance term in (26)	-12.7%	-12.1%	-10.2%	-8.5%
Net TFP loss from Markup Dispersion in (26)	3.8%	4.3%	5.4%	6.3%
% of TFP Loss from Markup Dispersion Mitigated: $-\frac{\text{Covariance Term in (26)}}{\text{Markup Dispersion Term in (26)}}$	76.9%	73.8%	65.4%	57.5%
Dispersion of Constraint Tightness Term in (26)	8.5%	7.8%	7.0%	6.5%

Notes. The table shows how TFP losses and their decompositions based on (26) vary with ϕ^A. When we vary ϕ^A, we recalibrate β while keeping all other parameters fixed, so that aggregate saving $\mathbb{E}[a]$ remains fixed.

from markup dispersion represented by the first term in (26) is higher. However, markups are also more negatively correlated with constraint tightness, in that $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ is more negative (Fact 1b). Together, the latter effect dominates, so the net TFP loss from markup dispersion, which is the sum of the first and third terms in (26), becomes smaller, since the $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ term in (26) offsets a larger fraction of the TFP loss from markup dispersion.

Conversely, when ϕ^A is higher, firms rely less on earnings to borrow: $Var\left(\log\mu\right)$ is lower, but $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ is also less negative. Consequently, the net TFP loss from markup dispersion is higher, even though the gross TFP loss from markup dispersion is lower, since the third term in (26) offsets a smaller fraction of the loss.[22]

The possibility of borrowing against earnings is crucial for the borrowing constraint channel to mitigate the TFP loss from markup dispersion. Without the option of borrowing against earnings ($\phi^\pi = 0$), as shown in Table 9, $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ is less negative, and the covariance term provides a weaker offset to the TFP loss from markup dispersion. Specifically, in this case ($\phi^\pi = 0$), the TFP loss decomposition in (26) is:

$$\underbrace{15.3\%}_{\text{TFP loss}} \approx \underbrace{12.6\%}_{\text{markup dispersion}} + \underbrace{6.7\%}_{\text{dispersion in constraint tightness}} - \underbrace{4.0\%}_{\text{higher-markup firms less constrained}}. \tag{28}$$

The third term in (26) is now smaller and only offsets 31.6% of the TFP loss from markup dispersion: the net TFP loss from markup dispersion is now higher at 12.6% − 4.0% = 8.6%. Consequently, the net TFP loss from markup dispersion and the total TFP losses are higher than in (27).

[22]The dispersion of constraint tightness in (26) moderately decreases with ϕ^A, rendering the total TFP loss in Table 10 roughly constant across values of ϕ^A.

5.3 TFP Gains from Policies Designed to Eliminate Markup Distortions

With borrowing constraints, TFP gains from subsidies designed to remove markup distortions become smaller. Consider the following subsidies to the intermediate good producer that fully remove markup distortions in a setting without borrowing constraints (Edmond, Midrigan, and Xu, 2023):

$$T\left(y/Y;\lambda\right) \equiv \lambda\Upsilon\left(y/Y\right) - \lambda\Upsilon'\left(y/Y\right)y/Y = \lambda\Upsilon\left(y/Y\right) - p\left(y/Y\right)y, \tag{29}$$

where we use the demand curve (3) for the latter equality.[23] Given the subsidies in (29), the intermediate good producer's objective in (8) is then

$$\begin{aligned} n(a,z) \equiv \max_{k,l} \quad & p\left(y/Y\right)y + T\left(y/Y;\lambda\right) - wl - c - (k-a)(1+r) + k(1-\delta) \\ = \max_{k,l} \quad & \lambda\Upsilon\left(y/Y\right) - wl - c - (k-a)(1+r) + k(1-\delta), \end{aligned} \tag{30}$$

subject to (3), (6), and (9).[24] The subsidies in (29) allow the intermediate good producer to value output from the competitive final goods producer's perspective, $\lambda\Upsilon\left(y/Y\right)$, rather than from the perspective of its own revenue $p\left(y/Y\right)y$.

Without borrowing constraints, the intermediate good producer's optimal choice of labor l then implies

$$p\left(y/Y\right) = \frac{\lambda}{Y}\Upsilon'\left(y/Y\right) = \frac{w}{(1-\alpha)z(k/l)^{\alpha}} = MC. \tag{31}$$

Compared to (13), we can see that the subsidies eliminate markup distortion by making it optimal for every intermediate good producer to set its markup to be 1. As in Edmond, Midrigan, and Xu (2023), these subsidies can therefore generate substantial TFP gains by removing markup dispersion.

With borrowing constraints, however, the TFP gains from these subsidies are moderated for two reasons. First, the net TFP loss from markup dispersion is already lower, because of the negative correlation between markups and constraint tightness. Second, such policies no longer fully eliminate markup dispersion. When earnings-based borrowing constraints bind, the optimal labor choice also accounts for how labor affects earnings and hence borrowing capacity, so (31) no longer holds.[25]

[23] Here, λ corresponds to DY in Edmond, Midrigan, and Xu (2023).

[24] Note that the subsidies $T\left(y/Y;\lambda\right)$ do not directly enter the earnings in (10), which determine the capacity for borrowing against earnings, as captured by (9). This is because, in practice, such borrowing capacity is based on firms' EBITDA (earnings before interest, taxes, depreciation, and amortization), which is calculated before any taxes or subsidies are applied.

[25] The cancellation behind (13) does not apply here: as discussed in Footnote 24, the borrowing limit is tied to pre-subsidy earnings, but the objective includes the subsidy through (30).

Table 11: TFP Gains from Subsidies Designed to Eliminate Markup Distortions

	Baseline	No Constraint	Baseline	No Constraint
	Fixed $G(a,z)$		Endogenous $G(a,z)$	
TFP gain with subsidies (29)	6.0%	11.7%	7.1%	11.7%
Total TFP loss without subsidies (29)	12.0%	11.7%	12.0%	11.7%
Total TFP loss with subsidies (29)	6.0%	0.0%	5.0%	0.0%

Notes. The table shows the TFP gains from the subsidies (29). "Baseline" corresponds to the baseline calibration with $\phi^A = 0.2$. "No constraint" corresponds to the case without borrowing constraints ($\phi^A = \infty$), while holding other parameters fixed.

To illustrate this point, Table 11 reports TFP gains from subsidies (29), comparing our baseline calibration ($\phi^A = 0.2$) with the case without borrowing constraints ($\phi^A = \infty$). The first row reports the TFP gains from the subsidies, each measured as the reduction in total TFP losses. The second and third rows report the underlying total TFP losses without and with the subsidies, respectively, calculated from (27).

We find that the TFP gains from the subsidies are smaller with borrowing constraints (around 6-7%) than without borrowing constraints (11.7%). This result holds whether we fix the stationary distribution of firms, $G(a,z)$, at its pre-subsidy counterpart to isolate the allocative efficiency implications of the subsidies (first two columns), or allow the subsidies to influence saving behavior and, correspondingly, the stationary distribution $G(a,z)$ (last two columns). There are two reasons. First, the two key mechanisms behind the negative correlation between markups and constraint tightness make higher-markup firms less constrained, which mitigates TFP losses from markup dispersion even without the subsidies and renders the subsidies less effective. Second, the subsidies themselves do not fully eliminate markup dispersion.

In Appendix C, we also study two additional policies considered in Edmond, Midrigan, and Xu (2023). The first is a size-dependent subsidy that, without borrowing constraints, eliminates markup dispersion while keeping the average markup at its pre-subsidy level. The second is a uniform subsidy that, without borrowing constraints, reduces the average markup to 1 but leaves markup dispersion unchanged. Tables IA11 and IA12 show that the first policy continues to generate sizable TFP gains once borrowing constraints are introduced: by keeping the average markup in place, it sustains higher firm earnings, which relaxes their borrowing constraints and brings outcomes closer to the unconstrained benchmark. The second policy, by contrast, delivers much smaller TFP gains with borrowing constraints, since lowering the average markup to 1 erodes firms' earnings and tightens their borrowing constraints; this remains true even when we allow endogenous saving.

6 Conclusion

In this paper, we document new facts that link firms' markups to borrowing constraints: (1) less constrained firms have higher markups, especially in industries where assets are difficult to borrow against and firms rely more on earnings to borrow; (2) markup dispersion is also higher in industries where firms rely more on earnings to borrow. We explain these relationships using a standard Kimball demand model augmented with borrowing against assets and earnings. The key mechanisms are as follows. First, less constrained firms have lower marginal costs and, hence, higher market shares and markups. This applies to both asset-based and earnings-based borrowing constraints. Second, when firms rely on earnings to borrow, higher productivity firms tend to have looser constraints because of their higher earnings; these firms also tend to have higher markups. This channel is stronger in industries with more reliance on borrowing against earnings. These mechanisms help our model quantitatively explain the empirical facts. They also have important implications for allocative efficiency: borrowing constraints lower the overall TFP losses from markup dispersion, especially in industries that rely more on earnings to borrow.

In sum, borrowing constraints provide new insights for understanding the dispersion of markups and its macroeconomic impact. Our work complements studies that examine markup dispersion due to other features of production, products, or pricing (Aghion et al., 2025; Eeckhout and Veldkamp, 2023; Salgado et al., 2025; Bornstein and Peter, 2025; Pellegrino, 2025; Di Tella, Malgieri, and Tonetti, 2025). Future work can further investigate the potential interaction between financial constraints and various determinants of markup dispersion, or the interaction with firm growth through the life cycle (Sedláček and Sterk, 2017; Sterk, Sedláček, and Pugsley, 2021).

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Online Appendix: Borrowing Constraints, Markups, and Misallocation

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This Appendix contains further material for the article "Borrowing Constraints, Markups, and Misallocation." Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."—"B." refer to the main article.

A Supplementary Theoretical Details

We use superscript $*$ to denote the point around which we take an approximation. We approximate around a symmetric point where all intermediate goods producers have zero capital wedges $\tau_k^*(i) = 0$, charge a given constant markup $\mu^*(i) = \mu^*$, and have the same productivity $z^*(i) = z$ and hence market share ($\tilde{y}^*(i) = 1$). From intermediate goods producers' optimal choice of labor (13), we know that $p^*(i) = \mu^* \frac{w^{1-\alpha}(r+\delta)^\alpha}{(1-\alpha)^{1-\alpha}\alpha^\alpha}/z$. From the final good producers' problem (2), and its implied demand curve (3) we know that $P^* = \mu^* \frac{w^{1-\alpha}(r+\delta)^\alpha}{(1-\alpha)^{1-\alpha}\alpha^\alpha}\frac{1}{z}$ and $\tilde{\lambda}^* \equiv \frac{\lambda^*}{Y^*} = P^*$, where we use

$$\Upsilon'\left(\tilde{y}(i)\right) = \left[\left(1-\eta\right)\tilde{y}(i) + \eta\right]^{-\frac{1}{\sigma(1-\eta)}} \tag{A.1}$$

based on (4). We use a hat over a variable to denote its log deviation from the point of approximation, e.g., $\hat{p}(i) = \ln\left(\frac{p(i)}{p^*(i)}\right)$ and $\hat{\mu}(i) = \ln\left(\frac{\mu(i)}{\mu^*}\right)$, with the exception $\hat{\tau}_k(i) = \ln(1+\tau_k(i))$, to accommodate the fact that $\tau_k^*(i) = 0$.

Proof of Proposition 1. From the demand elasticity under Dotsey and King (2005)'s specification, we know that

$$\mu(i) = \frac{\sigma\left(\tilde{y}(i)\right)}{\sigma\left(\tilde{y}(i)\right)-1} = \frac{\sigma\left(\eta + \left(1-\eta\right)\tilde{y}(i)\right)}{\sigma\eta + \left(\sigma\left(1-\eta\right)-1\right)\tilde{y}(i)},$$

so, to first order,

$$\hat{\mu}(i) - \hat{\mu}\left(j\right) \approx \frac{\eta}{\sigma-1}\left(\hat{\tilde{y}}(i) - \hat{\tilde{y}}\left(j\right)\right).$$

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From (13) and (14), we know that

$$\hat{MC}(i) = \alpha\hat{\tau}_k(i) - \hat{z}(i) = \hat{p}(i) - \hat{\mu}(i).$$

From the demand curve (3), we know that, to first order,

$$\hat{p}(i) - \hat{\tilde{\lambda}} \approx -\frac{1}{\sigma}\hat{\tilde{y}}(i),$$

where $\tilde{\lambda} \equiv \frac{\lambda}{Y}$. Together, we have, to first order,

$$\begin{aligned} \hat{\tilde{y}}(i) - \hat{\tilde{y}}(j) &\approx -\sigma\left(\hat{\mu}(i) - \hat{\mu}(j) + \alpha\left(\hat{\tau}_k(i) - \hat{\tau}_k(j)\right) - \left(\hat{z}(i) - \hat{z}(j)\right)\right), \\ \hat{\mu}(i) - \hat{\mu}(j) &\approx -\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\left[\alpha\left(\hat{\tau}_k(i) - \hat{\tau}_k(j)\right) - \left(\hat{z}(i) - \hat{z}(j)\right)\right], \end{aligned} \tag{A.2}$$

which proves Proposition 1.

Proof of Proposition 2. Directly from (17).

Proof of Proposition 3. From (17), we have, to second order,

$$Var\left(\log\mu(i)\right) = Var\left(\hat{\mu}(i)\right) \approx \left(\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\right)^2 \left(\alpha^2 Var\left(\hat{\tau}_k(i)\right) + Var\left(\hat{z}(i)\right) - 2\alpha Cov\left(\hat{\tau}_k(i), \hat{z}(i)\right)\right).$$

Together with Proposition 2, we have, to second order,

$$Var\left(\hat{\mu}(i)\right) \approx \left(\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\right)^2 Var(\hat{z}(i)) - \alpha^2\left(\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\right)^2 Var(\hat{\tau}_k(i)) - 2\alpha\frac{\sigma\eta}{\sigma - 1 + \sigma\eta} Cov(\hat{\mu}(i), \hat{\tau}_k(i)).$$

Proof of Proposition 4. From (22), to first order,

$$\hat{\tilde{y}}^P(i) - \hat{\tilde{y}}^P(j) \approx \sigma\left(\hat{z}(i) - \hat{z}(j)\right)$$

From (23), to first order,

$$\hat{\tilde{y}}(i) - \hat{\tilde{y}}(j) \approx \sigma\left(\hat{z}(i) - \hat{z}(j) - \left(\hat{\mu}(i) - \hat{\mu}(j) + \alpha\left(\hat{\tau}_k(i) - \hat{\tau}_k(j)\right)\right)\right).$$

Because the Kimball aggregation implies $\int \hat{\tilde{y}}^P(j)\,dj = \int \hat{\tilde{y}}(j)\,dj \approx 0$ to first order, integrating the previous two equations over j leads to (24) to first order.

Proof of Proposition 5. We will use the property that, for any $\{\hat{x}(i)\}_{i\in[0,1]}$, to second order, we have

$$\begin{aligned} \ln\left(\int e^{\hat{x}(i)}di\right) &\approx \ln\left(\int\left(1 + \hat{x}(i) + \frac{1}{2}(\hat{x}(i))^2\right)di\right) \approx \int\left(\hat{x}(i) + \frac{1}{2}(\hat{x}(i))^2\right)di - \frac{1}{2}\left(\int \hat{x}(i)\,di\right)^2 \\ &= \mathbb{E}[\hat{x}(i)] + \frac{1}{2}\mathrm{Var}(\hat{x}(i)), \end{aligned} \tag{A.3}$$

where $\mathbb{E}[\cdot]$ and $\text{Var}(\cdot)$ are over i. From the intermediate good producers' technology (6) and the definition of capital wedge (14), we have

$$l(i) = \frac{y(i)}{z(i)}\left(\frac{k(i)}{l(i)}\right)^{-\alpha} = \frac{y(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{w}{(r+\delta)}\right)^{-\alpha}(1+\tau_k(i))^{\alpha}$$
$$k(i) = l(i)\frac{k(i)}{l(i)} = \frac{y(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{w}{(r+\delta)}\right)^{1-\alpha}(1+\tau_k(i))^{\alpha-1}$$

where $\tilde{\lambda} \equiv \frac{\lambda}{Y}$. From the definition of TFP in (25), we have

$$Z = \frac{1}{\left(\int \frac{\tilde{y}(i)}{z(i)}(1+\tau_k(i))^{\alpha}\,di\right)^{1-\alpha}\left(\int \frac{\tilde{y}(i)}{z(i)}(1+\tau_k(i))^{\alpha-1}\,di\right)^{\alpha}} \tag{A.4}$$

$$\hat{Z} = -(1-\alpha)\ln\left(\int e^{\hat{\tilde{y}}(i)-\hat{z}(i)+\alpha\hat{\tau}_k(i)}\,di\right) - \alpha\ln\left(\int e^{\hat{\tilde{y}}(i)-\hat{z}(i)+(\alpha-1)\hat{\tau}_k(i)}\,di\right). \tag{A.5}$$

From (2), (3), and (A.1), we have

$$\int \Upsilon\left(\Upsilon'^{-1}\left(\frac{p(i)}{\tilde{\lambda}}\right)\right)di = 1 \quad \text{where} \quad (\Upsilon')^{-1}(x) = \frac{x^{-\sigma(1-\eta)}-\eta}{1-\eta}.$$

Together with (4), this means

$$\tilde{\lambda} = \left(\int p(i)^{1-\sigma(1-\eta)}\,di\right)^{\frac{1}{1-\sigma(1-\eta)}}$$
$$\hat{\tilde{\lambda}} = \frac{1}{1-\sigma(1-\eta)}\ln\left(\int e^{(1-\sigma(1-\eta))\hat{p}(i)}\,di\right)$$

Together with (A.3), we have, to second order,

$$\hat{\tilde{\lambda}} \approx \mathbb{E}\left[\hat{p}(i)\right] - \frac{\sigma(1-\eta)-1}{2}\text{Var}\left(\hat{p}(i)\right).$$

From the demand curve (3) and the function form of Dotsey and King (2005) for $\Upsilon'(\cdot)$ in (A.1), we have

$$\tilde{y}(i) = -\frac{\eta}{1-\eta} + \frac{1}{1-\eta}\left(\frac{p(i)}{\tilde{\lambda}}\right)^{-\sigma(1-\eta)}.$$

To second order, we have

$$\begin{aligned}\hat{\tilde{y}}(i) &\approx -\sigma\left(\hat{p}(i)-\hat{\tilde{\lambda}}\right) - \frac{1}{2}\sigma^2\eta\left(\hat{p}(i)-\hat{\tilde{\lambda}}\right)^2 \\ &\approx -\sigma\left(\hat{p}(i)-\mathbb{E}\left[\hat{p}(i)\right]+\frac{\sigma(1-\eta)-1}{2}\text{Var}\left(\hat{p}(i)\right)\right) \\ &\quad -\frac{1}{2}\sigma^2\eta\left(\hat{p}(i)-\mathbb{E}\left[\hat{p}(i)\right]+\frac{\sigma(1-\eta)-1}{2}\text{Var}\left(\hat{p}(i)\right)\right)^2.\end{aligned} \tag{A.6}$$

As a result, to second order, we have

$$\mathbb{E}\left[\hat{\tilde{y}}(i)\right] \approx -\frac{\sigma(\sigma(1-\eta)-1)}{2}\text{Var}\left(\hat{p}(i)\right) - \frac{\sigma^2\eta}{2}\text{Var}\left(\hat{p}(i)\right) = -\frac{\sigma(\sigma-1)}{2}\text{Var}\left(\hat{p}(i)\right). \tag{A.7}$$

From the functional form of Dotsey and King (2005) for demand elasticity in (5) and a second order approximation, we obtain

$$\mu(i) = \frac{\sigma\left(\tilde{y}(i)\right)}{\sigma\left(\tilde{y}(i)\right)-1} = \frac{\sigma\left(\eta+(1-\eta)\,\tilde{y}(i)\right)}{\sigma\eta+\left(\sigma(1-\eta)-1\right)\tilde{y}(i)}$$

$$\hat{\mu}(i) \approx \frac{\eta}{\sigma-1}\hat{\tilde{y}}(i) + \frac{\eta\left(1+\sigma(2\eta-1)-\eta\right)}{2(\sigma-1)^2}\left(\hat{\tilde{y}}(i)\right)^2.$$

From (A.3) and (A.5), we know that, to second order,

$$\begin{aligned}
\hat{Z} &\approx -(1-\alpha)\left(\mathbb{E}\left[\hat{\tilde{y}}(i)-\hat{z}(i)+\alpha\hat{\tau}_k(i)\right]+\frac{1}{2}\mathrm{Var}\left(\hat{\tilde{y}}(i)-\hat{z}(i)+\alpha\hat{\tau}_k(i)\right)\right) \\
&\quad -\alpha\left(\mathbb{E}\left[\hat{\tilde{y}}(i)-\hat{z}(i)+(\alpha-1)\,\hat{\tau}_k(i)\right]+\frac{1}{2}\mathrm{Var}\left(\hat{\tilde{y}}(i)-\hat{z}(i)+(\alpha-1)\,\hat{\tau}_k(i)\right)\right) \\
&\approx -\mathbb{E}\left[\hat{\tilde{y}}(i)-\hat{z}(i)\right]-\frac{1}{2}\mathrm{Var}\left(\hat{\tilde{y}}(i)-\hat{z}(i)\right)-\frac{\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right) \\
&\approx \frac{\sigma(\sigma-1)}{2}\mathrm{Var}\left(\hat{p}(i)\right)-\frac{1}{2}\mathrm{Var}\left(\sigma\hat{p}(i)+\hat{z}(i)\right)-\frac{\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right),
\end{aligned}$$

where we use (A.6), (A.7), and the fact that $\mathbb{E}\left[\hat{z}(i)\right]=0$ based on (7) in the stationary distribution. From (13) and (14), we know that

$$\hat{p}(i) = \hat{\mu}(i)+\alpha\hat{\tau}_k(i)-\hat{z}(i).$$

As a result, to second order,

$$\begin{aligned}
\hat{Z} &\approx \frac{\sigma(\sigma-1)}{2}\mathrm{Var}\left(\hat{\mu}(i)+\alpha\hat{\tau}_k(i)-\hat{z}(i)\right)-\frac{1}{2}\mathrm{Var}\left(\sigma\left(\hat{\mu}(i)+\alpha\hat{\tau}_k(i)-\hat{z}(i)\right)+\hat{z}(i)\right)-\frac{\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right) \\
&\approx \frac{\sigma-1}{2}\mathrm{Var}\left(\hat{z}(i)\right)-\frac{\sigma}{2}\mathrm{Var}\left(\hat{\mu}(i)\right)-\frac{\sigma\alpha^2+\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right)-\sigma\alpha\mathrm{Cov}\left(\hat{\mu}(i),\hat{\tau}_k(i)\right). \qquad \text{(A.8)}
\end{aligned}$$

Now consider the planner's problem in (21). Let λ_Y^P be the Lagrange multiplier on $\int_0^1 \Upsilon\left(\frac{y^P(i)}{Y^P}\right)di = 1$, λ_L^P be the Lagrange multiplier on $L=\int l^P(i)\,di$, and λ_K^P the Lagrange multiplier on $K=\int k^P(i)\,di$. Planner optimality implies that

$$\lambda_Y^P\frac{1-\alpha}{Y^P}\Upsilon'\left(\frac{y^P(i)}{Y^P}\right)z(i)\cdot\left(\frac{l^P(i)}{k^P(i)}\right)^{-\alpha}=\lambda_L^P \quad \text{and} \quad \lambda_Y^P\frac{\alpha}{Y^P}\Upsilon'\left(\frac{y^P(i)}{Y^P}\right)z(i)\cdot\left(\frac{l^P(i)}{k^P(i)}\right)^{1-\alpha}=\lambda_K^P,$$

which means

$$\frac{l^P(i)}{k^P(i)}=\frac{(1-\alpha)\lambda_K^P}{\alpha\lambda_L^P} \quad \text{and} \quad z(i)\Upsilon'\left(\frac{y^P(i)}{Y^P}\right)=\frac{(\lambda_K^P)^\alpha(\lambda_L^P)^{1-\alpha}}{\lambda_Y^P\alpha^\alpha(1-\alpha)^{1-\alpha}}Y^P.$$

From technology (6), we have

$$l^P(i)=\frac{y^P(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{\lambda_L^P}{\lambda_K^P}\right)^{-\alpha} \quad \text{and} \quad k^P(i)=\frac{y^P(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{\lambda_L^P}{\lambda_K^P}\right)^{1-\alpha}.$$

From the definition of TFP in (25), we have

$$Z^P = \frac{1}{\int \frac{\tilde{y}^P(i)}{z(i)} di},$$

where $\left\{\tilde{y}^P(i) \equiv \frac{y^P(i)}{Y^P}\right\}_{i\in[0,1]}$ is uniquely pinned down by the conditions that $z(i)\Upsilon'\left(\tilde{y}^P(i)\right)$ is constant in i and $\int \Upsilon\left(\tilde{y}^P(i)\right) di = 1$. Such a solution coincides with the equilibrium TFP when all firms charge a constant markup $\mu(i) = \mu^*$ and $\tau_k(i) = 0$ for all i. This is because, if $\mu(i) = \mu^*$ and $\tau_k(i) = 0$ for all i, the equilibrium TFP in (A.4) is given by $\frac{1}{\int \frac{\tilde{y}(i)}{z(i)} di}$, where $\tilde{y}(i)$ is uniquely pinned down by the conditions that $z(i)\Upsilon'\left(\tilde{y}(i)\right)$ is constant in i (from (3), (13), and (14) when $\mu(i) = \mu^*$ and $\tau_k(i) = 0$ for all i) and $\int \Upsilon\left(\tilde{y}(i)\right) di = 1$. By (A.8), to second order, we henceforth have

$$\hat{Z}^P \approx \frac{\sigma - 1}{2}\text{Var}\left(\hat{z}(i)\right).$$

As a result, to second order, we have

$$\hat{Z}^P - \hat{Z} \approx \frac{\sigma}{2}\text{Var}\left(\hat{\mu}(i)\right) + \frac{\sigma\alpha^2 + \alpha(1-\alpha)}{2}\text{Var}\left(\hat{\tau}_k(i)\right) + \sigma\alpha\text{Cov}\left(\hat{\mu}(i), \hat{\tau}_k(i)\right),$$

which proves Proposition 5.

Capital wedge and the Lagrange multiplier. We derive (15) and show that τ_k is strictly increasing in the Lagrange multiplier $\lambda_b \geq 0$ on the borrowing constraint (9), regardless of which borrowing constraint binds. The Lagrangian of the producer's static problem (8) is

$$\mathcal{L} = p(y/Y)y - wl - c - (k-a)(1+r) + k(1-\delta) + \lambda_b(\bar{b} - (k-a)),$$

where $\bar{b} \equiv \max\{\phi^A k, \phi^\pi \pi\}$ is the borrowing limit and $\lambda_b \geq 0$ ($\lambda_b = 0$ when the constraint is slack). Assuming the firm operates strictly in the interior of one constraint regime so that $\bar{b}$ is differentiable with respect to k, the first-order condition for l gives (13). This condition is unaffected by the borrowing constraint: for the asset-based constraint, the limit does not depend on l; for the earnings-based constraint, the multiplier factor scales both the marginal revenue and the variable cost of l proportionally and thus cancels out. Combining the first-order condition for k,

$$\left(p'(y/Y)\frac{y}{Y} + p(y/Y)\right)\alpha\frac{y}{k} = (r+\delta) + \lambda_b\left(1 - \frac{\partial\bar{b}}{\partial k}\right),$$

with the first-order condition for l and the definition of τ_k in (14) yields (15). We now verify that τ_k is strictly increasing in λ_b regardless of which borrowing constraint binds.

Asset-based constraint ($\bar{b} = \phi^A k$). Since $\frac{\partial \bar{b}}{\partial k} = \phi^A < 1$, equation (15) gives

$$\lambda_b = \frac{\tau_k(r+\delta)}{1-\phi^A},$$

which is linear and strictly increasing in τ_k.

Earnings-based constraint ($\bar{b} = \phi^\pi \pi$). Because the borrowing limit now depends on profits $\pi = p(y/Y)y - wl - c$, the Lagrangian becomes

$$\mathscr{L} = (1+\lambda_b\phi^\pi)(p(y/Y)y - wl - c) - k(r+\delta+\lambda_b) + a(1+r+\lambda_b).$$

The first-order condition for l remains (13): the factor $(1+\lambda_b\phi^\pi)$ scales both revenue and the variable cost of l, so it cancels. The first-order condition for k gives

$$(1+\lambda_b\phi^\pi)\left(p'(y/Y)\frac{y}{Y} + p(y/Y)\right)\alpha\frac{y}{k} = r+\delta+\lambda_b.$$

Combining with the first-order condition for l and the definition of τ_k in (14) yields $(1+\lambda_b\phi^\pi)(1+\tau_k)(r+\delta) = r+\delta+\lambda_b$, which simplifies to

$$\tau_k(r+\delta) = \frac{\lambda_b(1-\phi^\pi(r+\delta))}{1+\phi^\pi\lambda_b}.$$

Differentiating the right-hand side with respect to λ_b gives $\frac{1-\phi^\pi(r+\delta)}{(1+\phi^\pi\lambda_b)^2} > 0$ under the mild restriction $\phi^\pi(r+\delta) < 1$. This restriction is satisfied easily in our calibration: with $r = 0.02$, $\delta = 0.10$, and $\phi^\pi = 3.5$, we have $\phi^\pi(r+\delta) = 3.5 \times 0.12 = 0.42$. Hence τ_k is strictly increasing in λ_b. Inverting this relationship,

$$\lambda_b = \frac{\tau_k(r+\delta)}{1-\phi^\pi(r+\delta)(1+\tau_k)}.$$

B Additional Results

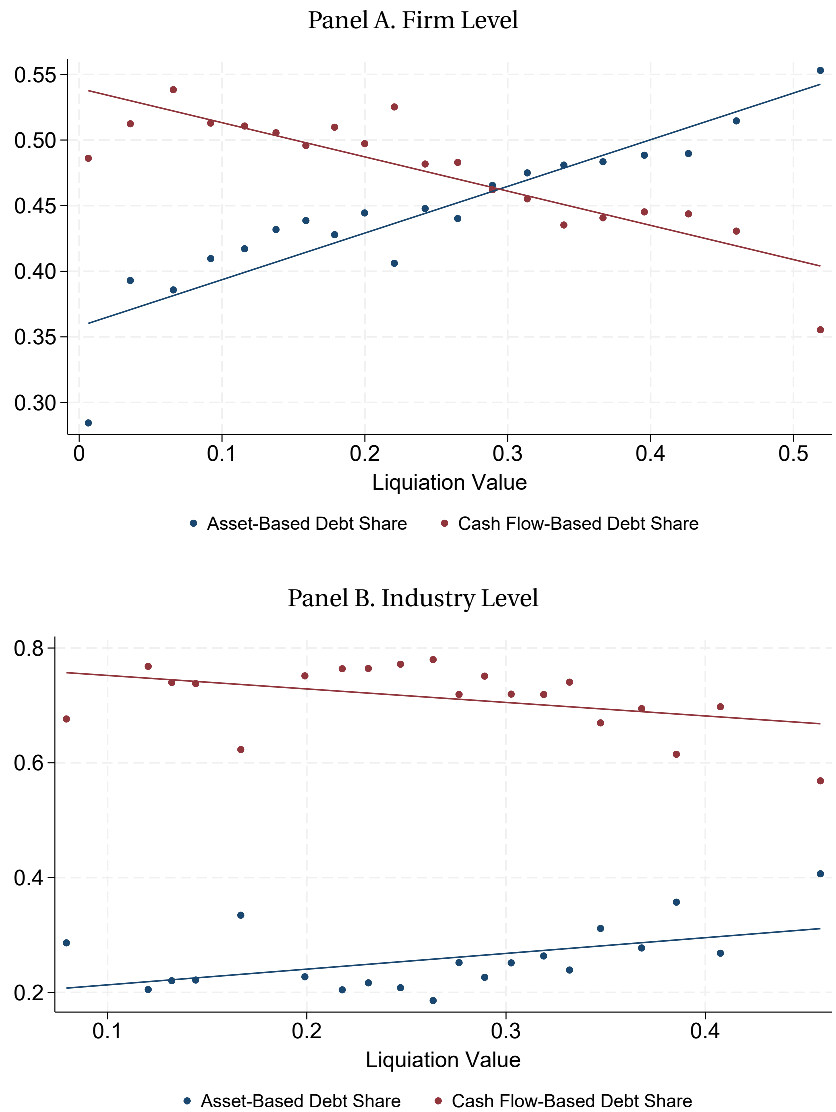

Figure IA1: Liquidation Value and Firm Borrowing. Panel A shows a binscatter plot of firm-year level share of debt that pledges the liquidation value of assets (asset-based debt) and debt that pledges earnings (cash flow-based debt) in total debt (measured using CapitalIQ data available from 2002 following Lian and Ma (2021)), as a function of the firm's liquidation value of fixed assets and working capital. Firm-level liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a), normalized by book assets. Panel B shows a binscatter plot of industry-year level share of asset-based debt and cash flow-based debt in total debt (measured using CapitalIQ data available from 2002 following Lian and Ma (2021)) against the industry's liquidation value of fixed assets and working capital. Industry-level debt share is calculated using total asset-based debt and cash flow-based debt in the industry divided by total debt in the industry. Industry-level liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a), normalized by total book assets.

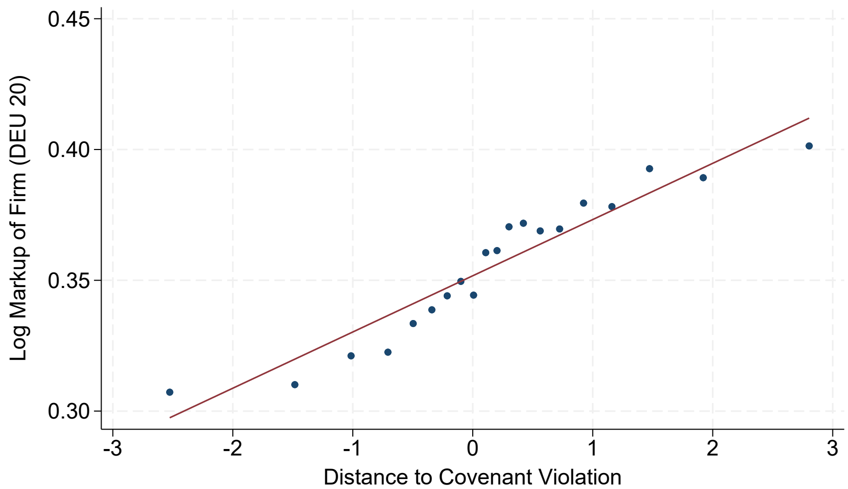

Figure IA2: Firm Markups and Constraint Tightness: Alternative Measure. This figure shows a binscatter plot of the relationship between log markup and firms' distance to violating earnings-based financial covenants, using U.S. Compustat firms annually from 1996 (beginning of DealScan data) to 2016 that have outstanding DealScan loans with earnings-based financial covenants. The x-axis represents firms' distance to violation, which is the standard deviation to violating at least one earnings-based covenant using covenant thresholds in DealScan, and the y-axis represents the log firm-level markup from De Loecker, Eeckhout, and Unger (2020). Industry (3-digit NAICS code) by year fixed effects are included. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

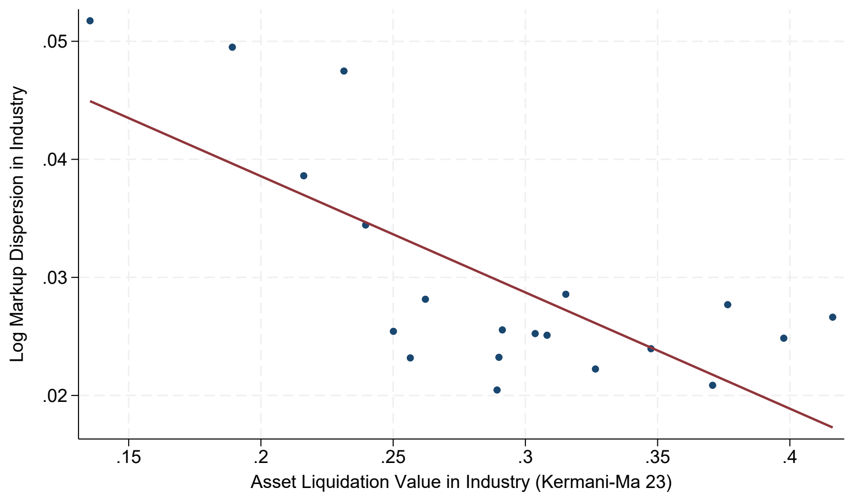

Figure IA3: Industry Markup Dispersion (Census of Manufactures). This figure shows a binscatter plot of the relationship between the variance of firm-level log markup (based on sales over intermediate inputs plus wage bill in Census of Manufactures data) in each industry-year on the y-axis, and the industry asset liquidation value from Figure 2 on the x-axis. Year fixed effects are included. We exclude observations with markup less than one.

Table IA1: Firm-Level Markups and Constraint Tightness, Controlling for Firm Age

	Firm Log Markup			
	(1)	(2)	(3)	(4)
	DEU	DEU w/	DEU	Translog
Markup Measure	Baseline	Overhead	Accounting	
Cash/Assets	0.389***	0.335***	0.191***	0.312***
	(0.034)	(0.029)	(0.017)	(0.048)
Log Firm Age	-0.011**	-0.013***	-0.007**	-0.002
	(0.004)	(0.004)	(0.002)	(0.004)
Observations	91,711	62,207	47,084	74,781
R^2	0.30	0.26	0.20	0.30

Notes. This table shows firm-year level regressions $\text{Log Markup}_{it} = \alpha_{ind(i)t} + \beta \text{Cash/Assets}_{it} + \gamma \text{Log Age}_{it} + \varepsilon_{it}$, using U.S. Compustat firms annually from 1987 to 2016. Log Markup_{it} for firm i in year t uses the baseline markup from De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input from De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup from De Loecker, Eeckhout, and Unger (2020) in column (3), and the translog markup following De Ridder (2024) in column (4). Compustat does not have a direct measure of firm age. We calculate age using the larger of time since incorporation (we collect incorporation year data from Datastream) and time since IPO (IPO year is from Compustat). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA2: Firm-Level Markups and Constraint Tightness: Alternative Measure

	Firm Log Markup			
	(1)	(2)	(3)	(4)
	DEU	DEU w/	DEU	Translog
Markup Measure	Baseline	Overhead	Accounting	
Distance to Covenant Violation	0.021***	0.017***	0.010***	0.021***
	(0.003)	(0.004)	(0.002)	(0.003)
Observations	25,226	16,261	13,076	23,581
R^2	0.34	0.33	0.24	0.36

Notes. This table shows firm-year level regressions Log $\text{Markup}_{it} = \alpha_{ind(i)t} + \beta\text{Distance to Violation}_{it} + \varepsilon_{it}$, using U.S. Compustat firms annually from 1996 (beginning of DealScan data) to 2016 that have outstanding DealScan loans with earnings-based financial covenants. Distance to violation is the standard deviation to violating at least one earnings-based covenant, using covenant thresholds in DealScan. Log Markup_{it} for firm i in year t uses the baseline markup from De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input from De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup from De Loecker, Eeckhout, and Unger (2020) in column (3), and translog markup following De Ridder (2024) in column (4). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA3: Relationship with Borrowing against Earnings

Panel A. Covariance between Markups and Cash/Assets

	Industry Cov(Log Markup, Cash/Assets)			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Cash Flow-Based Debt Share	0.008**	0.008***	0.008**	0.004**
	(0.003)	(0.003)	(0.003)	(0.002)
Observations	1,145	1,086	1,080	1,062
R^2	0.02	0.03	0.01	0.01

Panel B. Markup Dispersion

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Cash Flow-Based Debt Share	0.056**	0.025**	0.051**	0.031**
	(0.020)	(0.010)	(0.018)	(0.014)
Observations	1,146	1,091	1,087	1,064
R^2	0.04	0.03	0.03	0.03

Notes. Panel A shows industry-year level regressions Cov(Log Markup, Cash/Assets)$_{kt} = \alpha_t + \beta X_{kt} + \varepsilon_{kt}$, using U.S. Compustat firms annually from 2002 (beginning of cash flow-based debt data from CapitalIQ) to 2016. Cov(Log Markup, Cash/Assets)$_{kt}$ is the covariance of firm-level log markup and cash/assets among firms in industry k in year t. Firm-level markup uses the baseline markup from De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input from De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup from De Loecker, Eeckhout, and Unger (2020) in column (3), and translog markup following De Ridder (2024) in column (4). The variable X_{kt} is the fraction of debt that pledges earnings (cash flow-based debt) in total debt in industry k in year t; cash flow-based debt is measured using CapitalIQ data constructed following Lian and Ma (2021). Panel B shows industry-year level regressions Var(Log Markup)$_{kt} = \alpha_t + \beta X_{kt} + \varepsilon_{kt}$, among U.S. Compustat firms. Var(Log Markup)$_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Panel A. The variable X_{kt} is the fraction of debt that pledges earnings (cash flow-based debt) in total debt in industry k in year t. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA4: Industry Stock Return Volatility

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.216***	-0.161***	-0.091***	-0.118***
	(0.047)	(0.038)	(0.019)	(0.027)
Industry Stock Price Volatility	0.026*	0.019	0.009	0.004
	(0.015)	(0.013)	(0.007)	(0.011)
Fixed Effects	Year			
Observations	2,225	2,148	2,173	2,153
R^2	0.21	0.15	0.13	0.16

Notes. This table shows industry-year level regressions $\text{Var(Log Markup)}_{kt} = \alpha_t + \beta_1 \text{Liqval}_{kt} + \beta_2 \text{RetVol}_{kt} + \varepsilon_{kt}$, using U.S. Compustat firms annually from 1987 to 2016. $\text{Var(Log Markup)}_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Table 1. The variable Liqval_{kt} is the industry's liquidation value from fixed assets and working capital normalized by the industry's total book assets, as in Table 2. The variable RetVol_{kt} is the average firm-level stock return volatility in industry k in year t. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA5: Industry EBITDA/Debt Dispersion

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.203***	-0.161***	-0.088***	-0.109***
	(0.046)	(0.037)	(0.019)	(0.028)
Industry EBITDA/Debt Dispersion	0.000*	0.000	0.000**	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Fixed Effects	Year			
Observations	2,340	2,220	2,264	2,256
R^2	0.19	0.14	0.13	0.14

Notes. This table follows Table 2 with an additional control for industry EBITDA/debt dispersion. It shows industry-year level regressions $\text{Var(Log Markup)}_{kt} = \alpha_t + \beta_1 \text{Liqval}_{kt} + \beta_2 \text{Var(EBITDA/Debt)}_{kt} + \varepsilon_{kt}$, using U.S. Compustat firms annually from 1987 to 2016. $\text{Var(Log Markup)}_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Table 1. The variable Liqval_{kt} is the industry's liquidation value from fixed assets and working capital normalized by the industry's total book assets, as in Table 2. The variable $\text{Var(EBITDA/Debt)}_{kt}$ is the variance of firm-level EBITDA to debt ratio in industry k in year t. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA6: Census Markup and Compustat Markup

	Log Markup (Compustat, DEU)	
	(1)	(2)
Log Markup (Census)	0.396***	
	(0.062)	
Log Markup (Census, Translog)		0.396***
		(0.060)
Fixed Effects	Industry × Year	
Observations	7,300	7,300
R^2	0.28	0.28

Notes. This table shows firm level regressions Log Markup in $\text{Compustat}_{it} = \alpha_{ind(i)t} + \beta\text{Log Markup in Census}_{it} + \varepsilon_{it}$, for Compustat firms matched to Census of Manufactures. Log Markup in Compustat_{it} is the log markup in Compustat data following De Loecker, Eeckhout, and Unger (2020). Log Markup in Census_{it} for firm i in year t uses log markup based on sales over intermediate inputs plus wage bill in column (1) and log translog markup following De Ridder (2024) in column (2). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA7: Industry-Level Leverage and Asset Liquidation Value

	Industry Total Debt/Assets		Industry Total Debt/EBITDA	
	(1)	(2)	(3)	(4)
Industry Coverage	All	Manufacturing	All	Manufacturing
Industry Asset Liquidation Value	-0.065	0.165	-3.973*	5.350
	(0.100)	(0.273)	(2.031)	(3.179)
Fixed Effects	Year			
Observations	2,524	614	2,523	614
R^2	0.03	0.06	0.01	0.06

Notes. This table shows industry-year level regressions $\text{Leverage}_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, using U.S. Compustat firms annually from 1987 to 2016. Leverage_{kt} is total debt divided by total assets in industry k in year t in columns (1) and (2), and total debt divided by total EBITDA in columns (3) and (4). The regression uses all industries in columns (1) and (3), and manufacturing in columns (2) and (4). The variable Liqval_{kt} is the industry's asset liquidation value from fixed assets and working capital normalized by the industry's total book assets, as in Table 2. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA8: Measurement Error Checks

	Log Markup (Compustat, DEU)		
	(1)	(2)	(3)
Log Markup (Census)	0.592**	0.208**	0.263***
	(0.197)	(0.074)	(0.061)
Log Markup (Census) × Industry Asset Liquidation Value	-0.767		
	(0.577)		
Log Markup (Census) × Markup Dispersion (Census)		4.014	
		(2.008)	
Log Markup (Census) × Markup Dispersion (Compustat)			2.376
			(1.539)
Fixed Effects		Industry × Year	
Observations	7,300	7,300	7,300
R^2	0.29	0.29	0.29

Notes. This table shows firm level regressions Log Markup in Compustat$_{it} = \alpha_{ind(i)t} + \beta_1$Log Markup in Census$_{it}$ + β_2Log Markup in Census$_{it} \times Z_{ind(i)t} + \varepsilon_{it}$, for Compustat firms matched to the Census of Manufactures. Log Markup in Compustat$_{it}$ is the log markup for firm i in year t in Compustat data following De Loecker, Eeckhout, and Unger (2020). Log Markup in Census$_{it}$ for firm i in year t uses log markup based on sales over intermediate inputs plus wage bill in Census data. Industry characteristic $Z_{ind(i)t}$ is the asset liquidation value in firm i's industry in column (1), the dispersion of log markup in Census data in column (2), and the dispersion of log markup in Compustat data in column (3). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA9: Untargeted Moments and Model Counterparts

Untargeted moment	Source	Data	Model
Persistence of log markups $\log(\mu)$	Compustat	0.90	0.79
Variance of log net worth $Var(\log(a))$	Compustat	4.34	3.67
Bottom 10% asset (k) share	SOI	0	0.6%
Bottom 10% sales ($p \cdot y$) share	SOI	0	1.5%
Bottom 10% employment (l) share	BDS	0	2.4%

Notes. The table compares empirical moments not included in the calibration targets with the corresponding values generated by our model. Net worth (a in the model) corresponds to book equity in the data. The persistence and variance moments are calculated using Compustat data from 1987 to 2016. The bottom decile asset and sales shares are from Statistics of Income (SOI) from 1987 to 2016. The employment share is from the Business Dynamics Statistics (BDS) from the U.S. Census Bureau from 1987 to 2016.

Table IA10: Alternative Values of Variance of the Idiosyncratic Productivity Shock σ_e^2

	Baseline $\sigma_e^2 = 0.1$	Higher $\sigma_e^2 = 0.2$
Slope of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ w.r.t. ϕ^A	0.11	0.16
Slope of $Var\left(\log\mu\right)$ w.r.t. ϕ^A	-0.03	-0.05

Notes. The table displays results for alternative values of variance of the idiosyncratic productivity shock, σ_e^2, while keeping other parameters fixed.

C Additional Policy Results

This appendix reports TFP gains of two policy experiments that parallel Table 11. Let $\mathcal{M}$ denote the pre-subsidy aggregate cost-weighted markup.

The first policy is the size-dependent subsidy in Edmond, Midrigan, and Xu (2023) that, without borrowing constraints, eliminates markup dispersion while leaving the aggregate markup level equal to $\mathcal{M}$:

$$T^{SD}\left(y/Y;\lambda\right) \equiv \frac{\lambda}{\mathcal{M}}\Upsilon\left(y/Y\right) - \lambda\Upsilon'\left(y/Y\right)y/Y = \frac{\lambda}{\mathcal{M}}\Upsilon\left(y/Y\right) - p\left(y/Y\right)y. \tag{C.1}$$

Without borrowing constraints, this policy implies $p\left(y/Y\right) = \mathcal{M}\cdot MC$ for all firms. It therefore removes markup dispersion while preserving the aggregate markup level. With a binding earnings-based borrowing constraint, however, as discussed in Footnote 24, the subsidy does not directly enter borrowing capacity; the multiplier on the earnings-based borrowing constraint remains in the labor optimality condition, so the policy need not fully eliminate markup dispersion. Despite that, Table IA11 shows that the TFP gains from this size-dependent subsidy remain sizable with borrowing constraints. In the baseline calibration, the gain is 10.8% when the stationary distribution $G(a,z)$ is fixed and 11.1% when saving and the stationary distribution respond endogenously, close to the 11.7% gain in the no-constraint benchmark. The reason is that preserving the aggregate markup at $\mathcal{M}$ sustains operating earnings (10), which relaxes earnings-based borrowing constraints and keeps allocations close to the no-constraint benchmark.

Table IA11: TFP Gains from Size-Dependent Subsidies

	Baseline	No Constraint	Baseline	No Constraint
	Fixed $G(a,z)$		Endogenous $G(a,z)$	
TFP gain with subsidy (C.1)	10.8%	11.7%	11.1%	11.7%
Total TFP loss without subsidy (C.1)	12.0%	11.7%	12.0%	11.7%
Total TFP loss with subsidy (C.1)	1.2%	0.0%	1.0%	0.0%

Notes. The table shows the TFP gains from the subsidy (C.1). "Baseline" corresponds to the baseline calibration with $\phi^A = 0.2$. "No constraint" corresponds to the case without borrowing constraints ($\phi^A = \infty$), while holding other parameters fixed.

The second policy is a uniform subsidy. Following Edmond, Midrigan, and Xu (2023), we set the subsidy rate to $\mathcal{M}-1$:

$$T^{U}\left(y/Y;\lambda\right) \equiv (\mathcal{M}-1)\,\lambda\Upsilon'\left(y/Y\right)y/Y = (\mathcal{M}-1)\,p\left(y/Y\right)y. \tag{C.2}$$

Without borrowing constraints, this policy reduces the average markup to 1 but leaves relative markups, and hence markup dispersion, unchanged. With borrowing constraints, however, the policy delivers only small TFP gains, because the lower average markup reduces operating earnings (10) and tightens earnings-based borrowing constraints. Table IA12 shows that the uniform subsidy generates essentially no TFP gain in the no-constraint benchmark and only a small gain in the baseline calibration when $G(a,z)$ is fixed. When the stationary distribution responds endogenously, the TFP gain is negative, as the reduction in operating earnings tightens borrowing constraints enough to raise the total TFP loss.

Table IA12: TFP Gains from Uniform Subsidies

	Baseline	No Constraint	Baseline	No Constraint
	Fixed $G(a,z)$		Endogenous $G(a,z)$	
TFP gain with subsidy (C.2)	0.5%	0.0%	-1.0%	0.0%
Total TFP loss without subsidy (C.2)	12.0%	11.7%	12.0%	11.7%
Total TFP loss with subsidy (C.2)	11.5%	11.7%	13.0%	11.7%

Notes. The table shows the TFP gains from the subsidy (C.2). "Baseline" corresponds to the baseline calibration with $\phi^A = 0.2$. "No constraint" corresponds to the case without borrowing constraints ($\phi^A = \infty$), while holding other parameters fixed.