Appendix: Confidence and the Propagation of Demand Shocks

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Appendix A: Proofs

Preliminary step: the steady state.

We first provide conditions for the existence of a deterministic steady state and characterize it. By a deterministic steady state we mean a situation in which there are no shocks and all variables ($c, y, l, k, q, w, \vartheta$, R, and u) remain constant. Also, for this step we work with the original variables (not log-deviations).

In a steady state, the optimal labor supply implies

$$w^* \left(c^*\right)^{-\frac{1}{\sigma}} = \left(n^*\right)^{\frac{1}{\nu}},$$

while the optimal labor demand implies

$$w^* = \frac{\alpha}{1+\mu}\frac{y^*}{l^*}.$$

Using goods and labor market clearing $y^* = c^*$ and $l^* = n^*$, we have

$$l^* = \left(\frac{\alpha}{1+\mu}\right)^{\frac{\nu}{1+\nu}} \left(y^*\right)^{\frac{\nu}{1+\nu}\frac{\sigma-1}{\sigma}}.$$

Using it to replace labor in the production function, we have

$$\left(y^*\right)^{1-\frac{\alpha\nu}{1+\nu}\frac{\sigma-1}{\sigma}} = \left(\frac{\alpha}{1+\mu}\right)^{\frac{\alpha\nu}{1+\nu}} \left(u^*k^*\right)^{1-\alpha},$$

which is equivalent to

$$y^* = \left(\frac{\alpha}{1+\mu}\right)^{\frac{\alpha}{1+\frac{1}{\nu}-\alpha\left(1-\frac{1}{\sigma}\right)}} \left(u^*k^*\right)^{\frac{(1-\alpha)\left(1+\frac{1}{\nu}\right)}{1+\frac{1}{\nu}-\alpha\left(1-\frac{1}{\sigma}\right)}}.$$

Optimal utilization implies

$$\frac{1-\alpha}{1+\mu}\frac{y^*}{u^*k^*} = \vartheta^*\delta'\left(u^*\right).$$

The evolution of the shadow value of the land implies

$$\vartheta^* = \left(R^*\right)^{-1}\left(\frac{(1-\alpha)}{1+\mu}\frac{y^*}{k^*} + \left(1-\delta\left(u^*\right)\right)\vartheta^*\right),$$

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Optimal consumption implies

$$1 = (\beta R^*)^{-\sigma}.$$

Together, this means that the steady state value u^* must satisfy

$$1 = \beta \left(\delta' (u^*) u^* + (1 - \delta (u^*))\right). \tag{52}$$

Further notice that the evolution of capital in a steady state implies

$$\delta (u^*) = 0. \tag{53}$$

As discussed in the main text, for a steady state to exist, one needs the solution to (52) also satisfies (53), which implies

$$\kappa \equiv \frac{\delta' (u^*) u^*}{1 - \delta(u^*)} = \frac{1 - \beta}{\beta}. \tag{54}$$

Note that the existence of such a steady state imposes a restriction on the economy's parameters. When this restriction is violated, the economy exhibits a balance-growth path in which c, y, l, k, q, w, and ϑ grow at constant (although not necessarily equal) rates, while R and u remain constant. We have verified that this possibility does not upset any of our results. But we ignore it here because it is an artifact of the exclusion of an investment margin: once we add this margin (Section 6.4), a no-growth steady state is guaranteed for any non-infinite adjustment cost to capital. The logic is the same as in the textbook RBC model: diminishing returns to capital guarantee that gross investment exceeds (respectively, falls short of) depreciation when k is below (respectively, above) its steady-state value.

Proof of Proposition 1.

(19) and (21) directly follow from the main text. To derive (20), we first aggregate (16) and (17):

$$\begin{aligned}
\vartheta_t &= y_t - (1 + \phi) u_t - k_t, \\
\vartheta_t &= -R_t + (1 - \beta) \mathbb{E}_t \left[y_{t+1} - k_{t+1} - u_{t+1}\right] + \beta \mathbb{E}_t \left[\vartheta_{t+1}\right].
\end{aligned}$$

Combining we get

$$y_t - (1 + \phi) u_t - k_t = -R_t + (1 - \beta) \mathbb{E}_t \left[y_{t+1} - k_{t+1} - u_{t+1}\right] + \beta \mathbb{E}_t \left[y_{t+1} - (1 + \phi) u_{t+1} - k_{t+1}\right].$$

Using the aggregate production in (19), the evolution of aggregate capital in (21), and the fact $\kappa \equiv \frac{\delta'(u^*)u^*}{1-\delta(u^*)} = \frac{1-\beta}{\beta}$ from (54), we arrive at (20).

Proof of Lemma 1.

Lemma 1 follows directly from household h's Euler equation,

$$c_t^h = -\sigma \left(\beta_t^h + R_{h,t}\right) + E_t^h \left[c_{t+1}^h\right],$$

and its log-linearized budget constraint,

$$\sum_{k=0}^{+\infty}\beta^k c_{t+k}^h = b_t^h + \sum_{k=0}^{+\infty}\beta^k y_{h,t+k}.$$

Note that the Euler condition holds despite the informational friction, because we are using the household's *own* expectation operator.

Proof of Proposition 2.

Proposition 2 follows directly from the derivation in the main text.

Proof of Proposition 3.

From the aggregate production in (19), we have

$$\mathbb{E}_t\left[\sum_{k=0}^{+\infty}\beta^k y_{t+k}\right] = (1-\tilde{\alpha})\,\mathbb{E}_t\left[\sum_{k=0}^{+\infty}\beta^k\left(u_{t+k}+k_{t+k}\right)\right].$$

Then use the evolution of aggregate capital in (21), we have

$$\mathbb{E}_t\left[\sum_{k=0}^{+\infty}\beta^k\left(u_{t+k}+k_{t+k}\right)\right] = \mathbb{E}_t\left[\sum_{k=0}^{+\infty}\beta^k u_{t+k} + \frac{1}{1-\beta}k_t - \kappa\sum_{k=0}^{+\infty}\frac{\beta^{k+1}}{1-\beta}u_{t+k}\right].$$

Finally, use the fact $\kappa \equiv \frac{\delta'(u^*)u^*}{1-\delta(u^*)} = \frac{1-\beta}{\beta}$ from (54), we arrive at (31).

Proof of Lemma 2.

The expression for $\mathcal{B}_t$ follows directly from the derivation in the main text. Now we turn to the expression for $\mathcal{G}_t$.

First notice that, as all past aggregate shocks and all past aggregate outcomes are common knowledge, future $\mathcal{B}_{t+k}$ and $\mathcal{G}_{t+k}$ are only functions of the future aggregate shock η_{t+k}. They are henceforth unpredictable in period t. That is, $E_t^h\left[\mathcal{B}_{t+k}\right] = E_t^h\left[\mathcal{G}_{t+k}\right] = 0$ for all h, t, and all $k \geq 1$. From Proposition 4, we then have, for all h, t, and all $k \geq 1$,

$$E_t^h\left[\tilde{y}_{t+k}\right] = \varsigma E_t^h\left[R_{t+k}\right] + \beta E_t^h\left[\tilde{y}_{t+k+1}\right] \tag{55}$$

$$E_t^h\left[\tilde{y}_{t+k}\right] = -\sigma E_t^h\left[R_{t+k}+\beta_{t+k}\right] + \beta E_t^h\left[\tilde{y}_{t+k+1}\right], \tag{56}$$

As a result, we have for all h, t, and all $k \geq 1$,

$$E_t^h\left[R_{t+k}\right] = -\frac{\sigma}{\sigma+\varsigma}E_t^h\left[\beta_{t+k}\right].$$

From the definition of $\mathcal{G}_t$ in (25), we then have

$$\begin{aligned}\mathcal{G}_t &= \frac{\sigma^2}{\sigma+\varsigma}\sum_{k=1}^{+\infty}\beta^k\left(\bar{E}_t\left[\beta_{t+k}\right] - \mathbb{E}_t\left[\beta_{t+k}\right]\right)\\ &= -\tfrac{\sigma^2}{\sigma+\varsigma}\tfrac{\beta\rho_\beta}{1-\beta\rho_\beta}\left(\beta_t - \bar{E}_t\left[\beta_t\right]\right) = \tfrac{\sigma^2}{\sigma+\varsigma}\tfrac{\beta\rho_\beta}{1-\beta\rho_\beta}\left(\eta_t - \bar{E}_t\left[\eta_t\right]\right).\end{aligned}$$

Proof of Proposition 4.

Proposition 4 follows directly from Proposition 1, Proposition 2, and the definition of $\tilde{y}_t$ in (33). Finally, the terminal condition follows from the transversality condition of the firm.

Proof of Proposition 5.

Proposition 5 follows directly from the derivation in the main text.

Proof of Proposition 6.

Proposition 6 follows directly from (33), (38), and (39).

Proof of Proposition 7.

First notice that, as all past aggregate shocks and all past aggregate outcomes are common knowledge, future $\mathcal{B}_{t+k}$ and $\mathcal{G}_{t+k}$ are only functions of the future aggregate shock η_{t+k}. They are henceforth unpredictable in period t. From Proposition 4, we then have, for all t and all $k \geq 1$,

$$\begin{aligned}\mathbb{E}_t\left[\tilde{y}_{t+k}\right] &= \varsigma\mathbb{E}_t\left[R_{t+k}\right] + \beta\mathbb{E}_t\left[\tilde{y}_{t+k+1}\right]\\ \mathbb{E}_t\left[\tilde{y}_{t+k}\right] &= -\sigma\mathbb{E}_t\left[R_{t+k} + \beta_{t+k}\right] + \beta\mathbb{E}_t\left[\tilde{y}_{t+k+1}\right].\end{aligned}$$

As a result, we have for all t and all $k \geq 1$,

$$\mathbb{E}_t\left[R_{t+k}\right] = -\frac{\sigma}{\sigma+\varsigma}\rho_\beta^k\beta_t \quad \text{and} \quad \mathbb{E}_t\left[\tilde{y}_{t+k}\right] = -\frac{\sigma\varsigma}{\sigma+\varsigma}\frac{\rho_\beta^k}{1-\rho_\beta\beta}\beta_t.$$

Together with Lemma 2 and Proposition 4, we have that AS and AD can be re-expressed as follows:

$$\tilde{y}_t = \varsigma R_t - \frac{\sigma\varsigma}{\sigma+\varsigma}\frac{\beta\rho_\beta}{1-\rho_\beta\beta}\beta_t, \tag{57}$$

$$\tilde{y}_t = -\sigma R_t - \frac{\sigma\varsigma}{\sigma+\varsigma}\frac{\beta\rho_\beta}{1-\rho_\beta\beta}\beta_t + \frac{1-\beta}{1-\beta\rho_\xi}\left(\tilde{y}_t - \bar{E}_t\left[\tilde{y}_t\right]\right) + \tfrac{\sigma^2}{\sigma+\varsigma}\tfrac{\beta\rho_\beta}{1-\beta\rho_\beta}\left(\eta_t - \bar{E}_t\left[\eta_t\right]\right) - \sigma\beta_t.$$

Together with Proposition 5, we have

$$\left(1 + \sigma\varsigma^{-1} - \frac{1-\beta}{1-\beta\rho_\xi}(1-\lambda)\right)\frac{\partial\tilde{y}_t}{\partial\eta_t} = \frac{\sigma}{1-\rho_\beta\beta}\left(1 + \tfrac{\sigma\rho_\beta}{\sigma+\varsigma}(1-\lambda)\right).$$

It follows that

$$\frac{\partial y_t}{\partial\eta_t} = \frac{\sigma\beta\varsigma}{1-\rho_\beta\beta}\frac{1 + \frac{\sigma\beta\rho_\beta}{\sigma+\varsigma}(1-\lambda)}{\varsigma + \sigma - \varsigma\frac{1-\beta}{1-\beta\rho_\xi}(1-\lambda)},$$

which proves Proposition 7. Moreover, from (57), we have

$$\frac{\partial R_t}{\partial\eta_t} = \varsigma^{-1}\left(\frac{1}{\beta}\frac{\partial y_t}{\partial\eta_t} - \rho_\beta\gamma\right) > 0, \tag{58}$$

with γ is defined in Proposition 7.

Proof of Proposition 8.

Consider household h. From (12) - (14), we know

$$w_{h,t} = \frac{1}{\nu+1} y_{h,t} + \frac{\nu}{\sigma(\nu+1)} c_t^h$$

Together with the fact the household's total income is $y_{h,t}$, we know that her knowledge of $(w_{h,t}, e_{h,t})$ is informationally equivalent to her knowledge of $y_{h,t}$. In other words, (8) is equivalent to

$$\mathcal{I}_t^h = \mathcal{I}_{t-1}^h \cup \left\{\beta_t^h\right\} \cup \left\{y_{h,t}, R_{h,t}, (p_{i,j,t})_{i\in[0,1],j\in[0,1]}\right\} \cup \left\{\varepsilon_{t-1}^{\beta}\right\}.$$

We then know that the household h has three sources of knowledge about the AD shock η_t, $\left(R_{h,t}, y_{h,t}, \beta_t^h\right)$. Since past aggregate shocks and outcomes are common knowledge, the household h's information about η_t are given by following three independent signals about η_t :

$$\frac{\partial R_t}{\partial \eta_t}\eta_t + \epsilon_{h,t}^R, \quad \frac{\partial y_t}{\partial \eta_t}\eta_t + \xi_{h,t}, \quad \text{and} \quad -\eta_t + \epsilon_t^{\beta,h}.$$

From Proposition 7 and (58), we know $\frac{\partial y_t}{\partial \eta_t} = \gamma m(\lambda, \rho_\xi, \rho_\beta)$ and $\frac{\partial R_t}{\partial \eta_t} = \varsigma^{-1}\gamma\left[\beta^{-1} m(\lambda, \rho_\xi, \rho_\beta) - \rho_\beta\right] > 0$. Based on the standard formula for combining multiple Gaussian signals, we have

$$\bar{E}_t[\eta_t] = \lambda\eta_t = \frac{\sigma_\beta^{-2} + \sigma_R^{-2}\left\{\varsigma^{-1}\gamma\left[\beta^{-1} m(\lambda, \rho_\xi, \rho_\beta) - \rho_\beta\right]\right\}^2 + \sigma_\xi^{-2}\left\{\gamma m(\lambda, \rho_\xi, \rho_\beta)\right\}^2}{\sigma_{\text{AD}}^{-2} + \sigma_\beta^{-2} + \sigma_R^{-2}\left\{\varsigma^{-1}\gamma\left[\beta^{-1} m(\lambda, \rho_\xi, \rho_\beta) - \rho_\beta\right]\right\}^2 + \sigma_\xi^{-2}\left\{\gamma m(\lambda, \rho_\xi, \rho_\beta)\right\}^2}\eta_t,$$

which leads to (42).

Then note the following three properties of (42). First, the LHS of (42) increases with λ while the RHS of (42) decreases with λ. Second, at $\lambda = 0$, the LHS of (42) is smaller than the RHS of (42). Third, at $\lambda = 1$, the LHS of (42) is larger than the RHS of (42). As a result, there is a unique solution of (42) in $(0, 1)$. This proves Proposition 8.

Proof of Proposition 9.

First, from (42), we can directly verify that λ is a decreasing function of the ratio $\sigma/\sigma_{\text{AD}}$ for any $\sigma \in \{\sigma_\beta, \sigma_R, \sigma_\xi\}$. Because $m(\lambda, \rho_\xi, \rho_\beta)$ decreases in λ, we know m^* necessarily increases in the ratio $\sigma/\sigma_{\text{AD}}$ for any $\sigma \in \{\sigma_\beta, \sigma_R, \sigma_\xi\}$.

Second, suppose that there exists $\rho_\xi < \tilde{\rho}_\xi$ such that m^* at ρ_ξ is larger than m^* at $\tilde{\rho}_\xi$. First, we know that the RHS (42) is larger at ρ_ξ that at $\tilde{\rho}_\xi$. Second, because $m(\lambda, \rho_\xi, \rho_\beta)$ increases with ρ_ξ and decreases in λ, a larger m^* means that the equilibrium value of λ is smaller at ρ_ξ that at $\tilde{\rho}_\xi$. This means the LHS of (42) is smaller at ρ_ξ that at $\tilde{\rho}_\xi$. This leads to a contradiction.

Third, suppose that there exists $\rho_\beta < \tilde{\rho}_\beta$ such that m^* at ρ_β is larger than m^* at $\tilde{\rho}_\beta$. First, we know that the RHS (42) is larger at ρ_β than at $\tilde{\rho}_\beta$. Second, because $m(\lambda, \rho_\xi, \rho_\beta)$ increases with ρ_β and decreases in λ, a larger m^* means that the equilibrium value of λ is smaller at ρ_β that at $\tilde{\rho}_\beta$. This means the LHS of (42) is smaller at ρ_β that at $\tilde{\rho}_\beta$. This leads to a contradiction.

Proof of Proposition 10.

Optimal local labor demand and supply in (12) - (13) remain to be true, but the local production is given by

$$q_{i,t} = A_t + (1-\alpha)(u_{i,t} + k_{i,t}) + \alpha l_{i,t}.$$

Imposing labor market clearing and aggregating, we arrive at (44).

Now, note that the optimal utilization in (16) and the asset pricing equation in (17) remain to be true. Aggregating and combine terms, we have

$$y_t - (1+\phi)\,u_t - k_t = -R_t + (1-\beta)\,\mathbb{E}_t\left[y_{t+1} - k_{t+1} - u_{t+1}\right] + \beta\mathbb{E}_t\left[y_{t+1} - (1+\phi)\,u_{t+1} - k_{t+1}\right].$$

Similar to (20) but using the aggregate production in (44), we arrive at (45). The evolution of capital in (46) is the same as (21).

The derivation of the AD in (47) is exactly the same as the main analysis.

Proof of Proposition 11.

First, let us provide the formula for $\mathcal{B}_t$ and $\mathcal{G}_t$ with aggregate technology shocks. We first notice that, with aggregate technology shocks, the intertemporal version of Hulten's theorem in (31) becomes

$$\sum_{t=0}^{+\infty} \beta^{t+k}\mathbb{E}_t\left[y_{t+k}\right] = \frac{1-\tilde{\alpha}}{1-\beta}k_t + \sum_{t=0}^{+\infty}\beta^{t+k}\mathbb{E}_t\left[A_{t+k}\right].$$

As a result, $\mathcal{B}_t$ defined in (28) is now given by

$$\begin{aligned}\mathcal{B}_t &= \frac{(1-\beta)}{\beta}\sum_{k=0}^{+\infty}\beta^k \int \left(E_t^h\left[A_{t+k} + \xi_{h,t+k}\right] - \mathbb{E}_t\left[A_{t+k}\right]\right) dh \\ &= \frac{1-\beta}{\beta\left(1-\beta\rho_\xi\right)}\left(y_t - \bar{E}_t\left[y_t\right]\right) + \frac{1-\beta}{\beta\left(1-\beta\rho_A\right)}\left(\bar{E}_t\left[A_t\right] - \mathbb{E}_t\left[A_t\right]\right).\end{aligned}$$

For $\mathcal{G}_t$, from (25), we have

$$\mathcal{G}_t \equiv -\sigma\sum_{k=1}^{+\infty}\beta^k\left(\bar{E}_t\left[R_{t+k}\right] - \mathbb{E}_t\left[R_{t+k}\right]\right).$$

We now again define $\tilde{y}_t \equiv \frac{1}{\beta}\left(y_t - (1-\tilde{\alpha})k_t\right)$, similar to (33). Similar to Proposition 4, the AS and AD in Proposition 10 can be re-written as

$$\tilde{y}_t = \left[\varsigma\left(1-\rho_A\right) + \frac{1-\beta\rho_A}{\beta}\right]A_t + \varsigma R_t + \beta\mathbb{E}_t\left[\tilde{y}_{t+1}\right], \tag{59}$$

$$\tilde{y}_t = -\,\sigma R_t + \frac{1-\beta}{\beta}A_t + \beta\mathbb{E}_t\left[\tilde{y}_{t+1}\right] + \left(\mathcal{B}_t + \mathcal{G}_t\right). \tag{60}$$

From the above two equations, and similar to (55) and (56) in the proof of Lemma 2, we have, for all h, t, and all $k \geq 1$,

$$\begin{aligned}E_t^h\left[\tilde{y}_{t+k}\right] &= \left[\varsigma\left(1-\rho_A\right) + \frac{1-\beta\rho_A}{\beta}\right]E_t^h\left[A_{t+k}\right] + \varsigma E_t^h\left[R_{t+k}\right] + \beta E_t^h\left[\tilde{y}_{t+k+1}\right], \\ E_t^h\left[\tilde{y}_{t+k}\right] &= -\sigma E_t^h\left[R_{t+k}\right] + \frac{1-\beta}{\beta}E_t^h\left[A_{t+k}\right] + \beta E_t^h\left[\tilde{y}_{t+k+1}\right].\end{aligned}$$

As a result, we have for all h, t, and all $k \geq 1$,

$$E_t^h\left[R_{t+k}\right] = -E_t^h\left[\frac{\varsigma+1}{\sigma+\varsigma}\left(1-\rho_A\right)A_{t+k}\right]. \tag{61}$$

We can then rewrite $\mathcal{B}_t$ and $\mathcal{G}_t$ as

$$\mathcal{B}_t = \frac{1-\beta}{1-\beta\rho_\xi}\left(\tilde{y}_t - \bar{E}_t\left[\tilde{y}_t\right]\right) + \frac{1-\beta}{\beta\left(1-\beta\rho_A\right)}\left(\bar{E}_t\left[A_t\right]-A_t\right) \tag{62}$$

$$\mathcal{G}_t = \frac{\sigma\left(\varsigma+1\right)}{\sigma+\varsigma}\frac{\left(1-\rho_A\right)\beta\rho_A}{1-\beta\rho_A}\left(\bar{E}_t\left[A_t\right]-A_t\right). \tag{63}$$

Similar to (61), we have $\mathbb{E}_t\left[R_{t+k}\right] = -\mathbb{E}_t\left[\frac{\varsigma+1}{\sigma+\varsigma}\left(1-\rho_A\right)A_{t+k}\right]$ for all t and $k \geq 1$. From (59), we then have

$$\begin{aligned}
\mathbb{E}_t\left[\tilde{y}_{t+1}\right] &= \left[\frac{\varsigma\left(1-\rho_A\right)}{1-\beta\rho_A} + \frac{1}{\beta} - \frac{\varsigma\left(\varsigma+1\right)}{\sigma+\varsigma}\frac{\left(1-\rho_A\right)}{1-\beta\rho_A}\right]\rho_A A_t \\
&= \left[\frac{1}{\beta} + \frac{\varsigma\left(\sigma-1\right)}{\sigma+\varsigma}\frac{\left(1-\rho_A\right)}{1-\beta\rho_A}\right]\rho_A A_t.
\end{aligned}$$

We can then rewrite (59) as

$$\tilde{y}_t = \left[\varsigma\left(1-\rho_A\right)\left(1 + \frac{\left(\sigma-1\right)}{\sigma+\varsigma}\frac{\beta\rho_A}{1-\beta\rho_A}\right) + \frac{1}{\beta}\right]A_t + \varsigma R_t$$

or

$$R_t = \varsigma^{-1}\tilde{y}_t - \left[\left(1-\rho_A\right)\left(1 + \frac{\left(\sigma-1\right)}{\sigma+\varsigma}\frac{\beta\rho_A}{1-\beta\rho_A}\right) + \frac{\varsigma^{-1}}{\beta}\right]A_t.$$

Substituting into (60), we have

$$\left(1+\sigma\varsigma^{-1}\right)\tilde{y}_t = \left[\sigma\left(\frac{\varsigma^{-1}}{\beta} + \frac{1-\rho_A}{1-\beta\rho_A}\right) + \frac{1-\beta}{\beta\left(1-\beta\rho_A\right)}\right]A_t + \left(\mathcal{B}_t + \mathcal{G}_t\right), \tag{64}$$

and henceforth

$$\gamma^A = \frac{\partial y_t}{\partial \eta_t^A} = \beta\frac{\partial \tilde{y}_t}{\partial \eta_t^A} = \frac{\sigma\left(\varsigma^{-1} + \frac{\beta(1-\rho_A)}{1-\beta\rho_A}\right) + \frac{1-\beta}{1-\beta\rho_A}}{1+\sigma\varsigma^{-1}}.$$

Now we calculate $m_A^{\text{conf}}\left(\lambda, \rho_\xi, \rho_A\right)$ by using (62) and temporarily letting $\mathcal{G}_t = 0$. We have

$$\left(1+\sigma\varsigma^{-1}\right)\tilde{y}_t = \left[\sigma\left(\frac{\varsigma^{-1}}{\beta} + \frac{1-\rho_A}{1-\beta\rho_A}\right) + \frac{1-\beta}{\beta\left(1-\beta\rho_A\right)}\right]A_t + \frac{1-\beta}{1-\beta\rho_\xi}\left(\tilde{y}_t - \bar{E}_t\left[\tilde{y}_t\right]\right) + \frac{1-\beta}{\beta\left(1-\beta\rho_A\right)}\left(\bar{E}_t\left[A_t\right]-A_t\right).$$

As a result,

$$m_A^{\text{conf}}\left(\lambda, \rho_\xi, \rho_A\right) = \left(1 + \frac{\frac{(1-\beta)}{1-\beta\rho_\xi}\left(1-\lambda\right)}{1+\sigma\varsigma^{-1} - \frac{(1-\beta)}{1-\beta\rho_\xi}\left(1-\lambda\right)}\right)\left(1 - \frac{\frac{1-\beta}{1-\beta\rho_A}\left(1-\lambda\right)}{\sigma\left(\varsigma^{-1} + \frac{\beta(1-\rho_A)}{1-\beta\rho_A}\right) + \frac{1-\beta}{(1-\beta\rho_A)}}\right).$$

We then know that

$$m_A^{\text{conf}}\left(\lambda, \rho_\xi, \rho_A\right) < 1 \text{ if and only if } \rho_\xi < \bar{\rho}_\xi(\rho_A) \equiv \frac{1}{\beta} - \frac{\sigma\left(\varsigma^{-1}\left(1-\beta\rho_A\right) + \beta\left(1-\rho_A\right)\right) + \left(1-\beta\right)}{\beta\left(1+\sigma\varsigma^{-1}\right)},$$

where $\bar{\rho}_\xi(\rho_A) < \frac{1}{\beta}$ (so $\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_t\left[\xi_{i,t+k}\right]$ always converges) and increasing in ρ_A.

Now, we use (63) to replace $\mathcal{G}_t$ in (64) and have

$$m_A^{\text{GE}}\left(\lambda, \rho_A\right) = 1 - \frac{\frac{\sigma(\varsigma+1)}{\sigma+\varsigma}\frac{(1-\rho_A)\beta^2\rho_A}{1-\beta\rho_A}\left(1-\lambda\right)}{\sigma\left(\varsigma^{-1} + \frac{\beta(1-\rho_A)}{1-\beta\rho_A}\right) + \frac{1-\beta}{(1-\beta\rho_A)}\lambda} < 1.$$

Proof of Proposition 12.

As mentioned in the main text, here, we shut down the wealth effect on labor supply. As a result, the optimal labor supply in (13) becomes

$$n_t^i = \nu w_{i,t},$$

and $\tilde{\alpha}$ in aggregate production in (15) becomes

$$\tilde{\alpha} = 1 - \frac{(1-\alpha)\left(1+\frac{1}{\nu}\right)}{1+\frac{1}{\nu}-\alpha}.$$

With this new $\tilde{\alpha}$, the aggregate supply equations in Proposition 1 remains to be true.

In terms of aggregate demand, the individual optimal consumption in (22) becomes

$$c_t^h = (1-\beta)b_t^h - \beta\sigma\left\{\sum_{k=0}^{+\infty}\beta^k E_t^h\left[R_{h,t+k}\right]\right\} + (1-\beta)\left\{\sum_{k=0}^{+\infty}\beta^k E_t^h\left[y_{h,t+k} - T_{t+k}^h\right]\right\},$$

taking into consideration of the tax. Aggregating the above condition, using aggregate market clearing $y_t = c_t + G_t$ and the budget balance $G_t = T_t$, we have

$$\begin{aligned} y_t = & -\sigma\beta R_t - \beta\sigma\sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[R_{t+k}\right] + (1-\beta)\left(y_t + \sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[y_{t+k}\right]\right) \\ & + G_t - (1-\beta)\left(G_t + \sum_{k=1}^{+\infty}\beta^k\int E_t\left[G_{t+k}\right]dh\right) + \beta\mathcal{B}_t + \beta\mathcal{G}_t. \end{aligned}$$

where $\mathcal{B}_t$ and $\mathcal{G}_t$ are defined as (28) and (25). Rewriting this condition in recursive form and using (48), we reach at (49) in the main text:

$$y_t = -\sigma R_t + (1-\rho_G)\,G_t + \mathbb{E}_t\left[y_{t+1}\right] + \left(\mathcal{B}_t + \mathcal{G}_t\right).$$

In a nutshell, $(1-\rho_G)\,G_t$ replaces $-\sigma\beta_t$ as the AD shock in Proposition 2. The third part of Proposition 12 then follows directly from Proposition 7.

Proof of Proposition 13.

The aggregate supply block of the economy in Proposition 1 is unchanged and our intertemporal version of the Hulten's theorem in Proposition 3 is maintained. On the demand side, from the consumer optimality in Lemma 1 and similar to (29), we have

$$c_t^b = -\beta\sigma(R_t + \beta_t) - \beta\sigma\sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[R_{t+k} + \beta_{t+k}\right] + (1-\beta)\left(y_t + \sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[y_{t+k}\right]\right) + \beta\mathcal{B}_t + \beta\mathcal{G}_t. \quad (65)$$

$$c_t^s = -\beta\sigma R_t - \beta\sigma\sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[R_{t+k}\right] + (1-\beta)\left(y_t + \sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[y_{t+k}\right]\right) + \beta\mathcal{B}_t + \beta\mathcal{G}_t. \quad (66)$$

Together, the aggregate demand $y_t = \pi c_t^b + (1-\pi)\,c_t^s$ is given by

$$y_t = -\beta\sigma(R_t + \pi\beta_t) - \beta\sigma\sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[R_{t+k} + \pi\beta_{t+k}\right] + (1-\beta)\left(y_t + \sum_{k=1}^{+\infty}\beta^k\mathbb{E}_t\left[y_{t+k}\right]\right) + \beta\mathcal{B}_t + \beta\mathcal{G}_t,$$

where, similar to (25) and (32), $\mathcal{B}_t = \frac{1-\beta}{\beta(1-\beta\rho_\xi)}\left(y_t - \bar{E}_t\left[y_t\right]\right)$ and $\mathcal{G}_t = -\sigma\sum_{k=1}^{+\infty}\beta^k\left(\bar{E}_t\left[R_{t+k}\right] - \mathbb{E}_t\left[R_{t+k}\right]\right)$.

Together, this means that, the response of aggregate output to the AD shock is essentially the same as our benchmark economy, with aggregate discount factor now given by $\pi\beta_t$ and λ is still given by Proposition 5:

$$\frac{\partial y_t}{\partial \eta_t} = \pi\gamma \cdot m^{\text{conf}}\left(\lambda, \rho_\xi\right) \cdot m^{\text{GE}}\left(\lambda, \rho_\beta\right).$$

We now turn to saver's consumption. From (33), (66), Proposition 4, and Lemma 2, we have

$$\frac{\partial c_t^s}{\partial \eta_t} = -\varsigma^{-1}\sigma\frac{\partial y_t}{\partial \eta_t} + \tfrac{1-\beta}{1-\beta\rho_\xi}\left(1-\lambda\right)\frac{\partial y_t}{\partial \eta_t} + \beta\tfrac{\sigma^2}{\sigma+\varsigma}\tfrac{\beta\rho_\beta}{1-\beta\rho_\beta}\pi\left(1-\lambda\right).$$

As a result, $\frac{\partial c_t^s}{\partial \eta_t} > 0$ if and only if

$$\left[\tfrac{1-\beta}{1-\beta\rho_\xi}\left(1-\lambda\right) - \varsigma^{-1}\sigma\right]\gamma \cdot m^{\text{conf}}\left(\lambda, \rho_\xi\right) \cdot m^{\text{GE}}\left(\lambda, \rho_\beta\right) + \beta\tfrac{\sigma^2}{\sigma+\varsigma}\tfrac{\beta\rho_\beta}{1-\beta\rho_\beta}\left(1-\lambda\right) > 0. \tag{67}$$

A sufficient condition for the above is

$$\tfrac{1-\beta}{1-\beta\rho_\xi}\left(1-\lambda\right) - \varsigma^{-1}\sigma > 0. \tag{68}$$

That is, when the confusion of the savers is sufficiently large (λ small enough) and/or that the AS curve is sufficiently flat (ς large enough), the saver's consumption also positively comoves with the AD shock.

Similarly, for borrowers, from (33), (66), Proposition 4, and Lemma 2, we have

$$\frac{\partial c_t^b}{\partial \eta_t} = \sigma\left(\frac{\beta}{1-\rho_\beta\beta} - \varsigma^{-1}\frac{\partial y_t}{\partial \eta_t}\right) + \tfrac{1-\beta}{1-\beta\rho_\xi}\left(1-\lambda\right)\frac{\partial y_t}{\partial \eta_t} + \beta\tfrac{\sigma^2}{\sigma+\varsigma}\tfrac{\beta\rho_\beta}{1-\beta\rho_\beta}\pi\left(1-\lambda\right).$$

We then know that, if (67) or (68) is true, we also have $\frac{\partial c_t^b}{\partial \eta_t} > 0$. In sum, as long as (67) or (68) holds, c_t^s, c_t^b, and y_t comove.

Aggregate supply: the role of capital adjustment costs.

Here we verify the claim that, even with complete information, the combination of variable utilization and capital adjustment costs allows for an "upward-slopping AS curve," or for aggregate employment and output to comove with aggregate consumption in response to discount-rate shocks. In particular, we show that this is true for *any* $\psi > 0$, as long as the real wage is sufficiently acyclical, or equivalently when the wealth effect on labor supply is small enough and the Frisch elasticity is large enough.

Concretely, we consider the complete information version of the investment model in Proposition 14. We still assume away from wealth effect of labor supply, but allow a general AR(1) process for the idiosyncratic demand shock and allow the AD shock to be persistent. The complete information equilibrium can be solved based on a system of equations consisting of the aggregate production (15), the aggregate, complete-information version of (72), (73), (74), and (76), the aggregate Euler equation,

$$c_t = -\sigma\left(R_t + \beta_t\right) + \mathbb{E}_t\left[c_{t+1}\right],$$

and the aggregate resource constraint,

$$y_t = \frac{c^*}{y^*}c_t + \left(1 - \frac{c^*}{y^*}\right)\left(\iota_t + k_t\right).$$

Using Mathematica, we can solve the equilibrium policy rules in closed form for arbitrary parameters. Consider in particular the policy rule for utilization, which is the object of interest. This can be expressed as

$$u_t = \Lambda_{uk} k_t - \Lambda_{u\beta} \beta_t,$$

for some scalars $(\Lambda_{uk}, \Lambda_{u\beta})$ that are complicated functions of the underlying parameters. Taking the limit as labor supply becomes infinitely elastic, or wages become nearly acyclical, we obtain

$$\lim_{\nu \to +\infty} \Lambda_{u\beta} = \frac{\beta \frac{c^*}{y^*} \delta \sigma \psi}{-\beta^2 \frac{c^*}{y^*} \delta \rho_\beta \sigma \psi \phi + \beta(\phi(\frac{c^*}{y^*}(\delta \sigma \psi + \delta + \rho_\beta - 1) - \rho_\beta + 1) + \delta \psi (\frac{c^*}{y^*}(1-\delta) - \rho_\beta + 1)) + \frac{c^*}{y^*} \delta \psi}, \tag{69}$$

which is strictly positive for any $\psi > 0$ (and 0 for $\psi = 0$). We conclude that, as long as $\psi > 0$, a large enough ν, or a sufficiently acyclical real wage, suffices for aggregate employment and output to increase with η_t.

On the other hand, had we instead set $\psi = 0$ (no adjustment cost), as in the classics by Burnside, Eichenbaum, and Rebelo (1995), Greenwood, Hercowitz, and Huffman (1988) and King and Rebelo (1999), $\Lambda_{u\beta} = 0$ for *any* ν because utilization ceases to be forward-looking. It then follows from the intratemporal condition for labor, as in Barro and King (1984), that aggregate employment and output necessarily move in the opposite direction than aggregate consumption.

Proof of Proposition 14.

We first characterize the deterministic steady state. In this steady state, we have

$$\iota^* = \delta\left(u^*\right) \equiv \delta.$$

Optimal utilization implies

$$\frac{1-\alpha}{1+\mu} \frac{y^*}{u^*} = \vartheta^* \delta'\left(u^*\right) k^*.$$

The evolution of the shadow value of capital implies

$$\vartheta^* = \beta \left[\frac{(1-\alpha)}{1+\mu} \frac{y^*}{k^*} + (1-\delta) \vartheta^* \right].$$

Together, this means

$$1 = \beta \left[\delta'\left(u^*\right) u^* + (1-\delta) \right]. \tag{70}$$

Now, we turn to the analysis away from the steady state, in response to the aggregate demand shock. As mentioned in the main text, we let investment choices be made by the households and under the same information as consumption. The household h's information set in period t is given by $\mathcal{I}_t^h = \mathcal{I}_{t-1}^h \cup \left\{\beta_t^h\right\} \cup \left\{w_{h,t}, e_{h,t}, R_{h,t}, (p_{i,j,t})_{i \in [0,1], j \in [0,1]}, k_{h,t+1}\right\} \cup \left\{\eta_{t-1}\right\}.$

As in the baseline model, we log-linearize the equilibrium conditions, re-interpret all the variables as log-deviations from their steady-state counterparts.[1] As mentioned in the main text, we shut down the wealth effect on labor supply. We thus write labor supply (or the "wage equation") of the representative household on island i as

$$n_t^i = \nu w_{i,t}. \tag{71}$$

As in our analysis of government spending, the aggregate production in (15) continues to hold, with $\tilde{\alpha}$ redefined as follows:

$$\tilde{\alpha} = 1 - \frac{(1-\alpha)\left(1+\frac{1}{\nu}\right)}{1+\frac{1}{\nu}-\alpha}.$$

Similarly, the local firm's FOC for utilization is still given by (16), which is rewritten here:

$$y_{i,t} - u_{i,t} - k_{i,t} = \vartheta_{i,t} + \phi u_{i,t}. \tag{72}$$

The evolution of the shadow value of capital on island i is now given by[2]

$$\vartheta_{i,t} = -R_{i,t} + (1-\beta(1-\delta))\,\mathbb{E}_t\left[y_{i,t+1} - k_{i,t+1} - u_{i,t+1}\right] + \beta\mathbb{E}_t\left[\vartheta_{i,t+1}\right]. \tag{73}$$

Optimal investment by the representative household on island i is given by

$$0 = E_t^i\left[\vartheta_{i,t}\right] - \psi\delta\iota_t^i. \tag{74}$$

Optimal consumption by the representative household on island i is given by

$$c_{i,t} = -\sigma\left(\beta_t^i + R_{i,t}\right) + E_t^i\left[c_{i,t+1}\right]. \tag{75}$$

The evolution of local capital is given by

$$k_{i,t+1} = -\left(\delta + \frac{1-\beta}{\beta}\right)u_{i,t} + \delta\iota_t^i + \epsilon_{i,t}^k + k_{i,t}, \tag{76}$$

where we use (70).

To prove Proposition 14, we first take a detour and solve the complete-information policy rules of two different cases of our economy. They will be useful in the characterization of the incomplete information equilibrium later.

First, we solve the local equilibrium when the interest rate is fixed at the steady state value and the local demand is independent from the aggregate condition and is only given by the idiosyncratic shock

$$y_{i,t} = \xi_{i,t},$$

where $\xi_{i,t}$ follows a random walk ($\rho_\xi = 1$). Using (72), (73), (74), and (76) with $E_t^i[\cdot] = \mathbb{E}_t[\cdot]$, we can solve the local shadow value of capital as a function of the local capital stock and the local demand:

$$\vartheta_{i,t} = l_k k_{i,t} + l_\xi \xi_{i,t}. \tag{77}$$

---

[1] The only exception to this rule is that we let the new $\epsilon_{i,t}^k$ be the original $\epsilon_{i,t}^k$. This takes care of the issue that the steady-state value of $\epsilon_{i,t}^k$ is zero.

[2] (73) comes from the log-linearization of the following condition:

$$\vartheta_{i,t} = R_{i,t}^{-1}\mathbb{E}_t\left[\frac{(1-\alpha)}{1+\mu}\frac{y_{i,t+1}}{u_{i,t+1}k_{i,t+1}}u_{i,t+1} - \iota_{t+1}^i + \left(1-\delta\left(u_{i,t+1}\right)+\psi\left(\iota_{t+1}^i\right)\right)\vartheta_{i,t+1}\right].$$

Unfortunately, there are no *simple* closed-form formulas for l_k and l_ξ. However, using Mathematica, we are able first to obtain a complicated closed-form solution for arbitrary parameters and then to obtain the following simpler formulas for the case of a vanishing curvature in the utilization costs:

$$\lim_{\phi\to 0} l_k = -\frac{\psi\left(\beta^2\delta^2+\beta(2\delta-1)+1\right)}{(\beta\delta+1)(\beta(\delta-1)\psi+\beta+\psi)} \quad \text{and} \quad \lim_{\phi\to 0} l_\xi = \frac{\psi(\beta(\delta-1)+1)\left(\beta^2\delta^2+\beta(2\delta-1)+1\right)}{(\beta\delta+1)(\beta(\delta-\rho_\xi)+1)(\beta(\delta-1)\psi+\beta+\psi)}.$$

Now, we solve the complete-information policy rule of the aggregate economy without the aggregate demand shock. In particular, we consider a system of equations consists of the aggregate production (15), the aggregate, complete information, version of (72), (73), (74), and (76), the aggregate Euler equation without the aggregate demand shock

$$c_t = -\sigma R_t + \mathbb{E}_t\left[c_{t+1}\right],$$

and the aggregate market clearing $y_t = \frac{c^*}{y^*}c_t + \left(1-\frac{c^*}{y^*}\right)(\iota_t + k_t)$. We arrive at the following aggregate policy rule:

$$u_t = \Lambda_{uk}k_t \quad \text{and} \quad q_t = \Lambda_{qk}k_t. \tag{78}$$

Again, we use Mathematica to obtain Λ_{uk} and Λ_{qk} for arbitrary parameters and then take the limit as $\phi \to 0$ to get a somewhat more tractable solution.[3]

Now, we turn to the incomplete information economy. To prove Proposition 14, we impose three additional simplification restrictions: aggregate demand shock is i.i.d $(\rho_\beta = 0)$; the idiosyncratic demand shock is random walk ($\rho_\xi = 1$); and the information friction is large ($\lambda = 0$). That is, the typical household has no knowledge about the aggregate demand shock η_t and λ in Proposition 5 is zero. To prove Proposition 14, we only need to prove that, with those restrictions, there exists $\bar{\phi}, \underline{\nu}, \underline{\psi} > 0$ such that whenever $\nu > \underline{\nu}$, $\phi < \bar{\phi}$, and $\psi > \underline{\psi}$, the equilibrium levels of employment, output, investment, and consumption positively comove in response to the aggregate demand shock.

We want to characterize the equilibrium responses of the period-t outcomes to the concurrent AD shock, η_t. But since in the log-linearized economy the equilibrium response is independent from t, it suffices to characterize the equilibrium responses of the period-0 outcomes to η_0 (recall that the initial capital is fixed $k_0 = 0$).

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[3] As $\phi \to 0$, Λ_{uk} and Λ_{qk} converge to, respectively,

$$\begin{aligned}
&(-\gamma\delta\psi^2+\beta\gamma\delta\psi^2-\beta\gamma\delta^2\psi^2-\psi\tilde{\alpha}+\beta\psi\tilde{\alpha}+\gamma\psi\tilde{\alpha}-\beta\gamma\psi\tilde{\alpha}-2\beta\delta\psi\tilde{\alpha}+\delta\psi^2\tilde{\alpha}-\beta\delta\psi^2\tilde{\alpha}+\beta\delta^2\psi^2\tilde{\alpha}-\gamma\delta\sigma\psi^2\tilde{\alpha}+\beta\gamma\delta\sigma\psi^2\tilde{\alpha}-2\beta\gamma\delta^2\sigma\psi^2\tilde{\alpha}+\\
&\quad 2\beta^2\gamma\delta^2\sigma\psi^2\tilde{\alpha}-\beta^2\gamma\delta^3\sigma\psi^2\tilde{\alpha}-2\beta\tilde{\alpha}^2+2\beta\gamma\tilde{\alpha}^2+2\beta\delta\psi\tilde{\alpha}^2-2\beta\gamma\delta\sigma\psi\tilde{\alpha}^2-2\beta^2\gamma\delta^2\sigma\psi\tilde{\alpha}^2+\\
&\quad \surd(-4\beta\tilde{\alpha}(\psi+\beta(-1+\delta)\psi+\beta\tilde{\alpha})(\delta\psi+(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha})(\gamma\delta\psi(1-\beta\delta\sigma\psi)+(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha})+\\
&\qquad (\gamma\delta\psi(\psi+\beta(-1+\delta)\psi)+\psi(1-\delta\psi+\beta^2\gamma(-2+\delta)\delta^2\sigma\psi+\gamma(-1+\delta\sigma\psi)+\beta(-1+\gamma+2\delta+\delta\psi-\delta^2\psi-\gamma\delta\sigma\psi+2\gamma\delta^2\sigma\psi))\tilde{\alpha}+\\
&\qquad\quad 2\beta(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha}^2)^2))/(2(\psi+\beta(-1+\delta)\psi+\beta\tilde{\alpha})(\delta\psi+(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha})),
\end{aligned}$$

and

$$\begin{aligned}
&-((\tilde{\alpha}(2\delta\psi^2-2\beta\delta\psi^2-\gamma\delta\psi^2+\beta\gamma\delta\psi^2+2\beta\delta^2\psi^2-\beta\gamma\delta^2\psi^2+\psi\tilde{\alpha}-\beta\psi\tilde{\alpha}-\gamma\psi\tilde{\alpha}+\beta\gamma\psi\tilde{\alpha}+2\beta\delta\psi\tilde{\alpha}-2\beta\gamma\delta\psi\tilde{\alpha}-\delta\psi^2\tilde{\alpha}+\beta\delta\psi^2\tilde{\alpha}-\beta\delta^2\psi^2\tilde{\alpha}+\\
&\qquad \gamma\delta\sigma\psi^2\tilde{\alpha}-\beta\gamma\delta\sigma\psi^2\tilde{\alpha}+2\beta\gamma\delta^2\sigma\psi^2\tilde{\alpha}+\beta^2\gamma\delta^3\sigma\psi^2\tilde{\alpha}+\\
&\qquad \surd(-4\beta\tilde{\alpha}(\psi+\beta(-1+\delta)\psi+\beta\tilde{\alpha})(\delta\psi+(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha})(\gamma\delta\psi(1-\beta\delta\sigma\psi)+(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha})+\\
&\qquad\quad (\gamma\delta\psi(\psi+\beta(-1+\delta)\psi)+\psi(1-\delta\psi+\beta^2\gamma(-2+\delta)\delta^2\sigma\psi+\gamma(-1+\delta\sigma\psi)+\beta(-1+\gamma+2\delta+\delta\psi-\delta^2\psi-\gamma\delta\sigma\psi+2\gamma\delta^2\sigma\psi))\tilde{\alpha}+\\
&\qquad\qquad 2\beta(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha}^2)^2)))/(2(\psi+\beta(-1+\delta)\psi+\beta\tilde{\alpha})(\delta\psi+(1-\delta\psi+\gamma(-1+\delta\sigma\psi+\beta\delta^2\sigma\psi))\tilde{\alpha})))\}
\end{aligned}$$

where γ in the formulas corresponds to $\frac{c^*}{y^*}$ in the paper.

From (72), (73), and (74), the household's optimal investment decision is given by:

$$\iota_{i,0} = \frac{1}{\psi\delta}\left(-R_{i,0} + (1-\beta(1-\delta))\frac{\phi}{\phi+1}E_{i,0}\left[y_{i,1}-k_{i,1}\right] + \left(\frac{(1-\beta(1-\delta))}{\phi+1}+\beta\right)E_{i,0}\left[\vartheta_{i,1}\right]\right).$$

Now, because the typical household has no knowledge about the aggregate demand shock. In her expectation, future local demand is as if given only by the idiosyncratic shock, e.g., $E_{i,0}\left[y_{i,1}\right] = E_{i,0}\left[\xi_{i,1}\right]$. Then because the current period shocks become common knowledge next period, in the typical household's expectation, future shadow value of land can be characterized by the complete information policy rule in (77):

$$E_{i,0}\left[\vartheta_{i,1}\right] = E_{i,0}\left[l_k k_{i,1} + l_\xi \xi_{i,1}\right].$$

We then have:

$$\iota_{i,0} = \frac{1}{\psi\delta}\left(-R_{i,0} + (1-\beta(1-\delta))\frac{\phi}{\phi+1}\left(E_{i,0}\left[\xi_{i,1}-k_{i,1}\right]\right) + \left(\frac{(1-\beta(1-\delta))}{\phi+1}+\beta\right)\left(E_{i,0}\left[l_k k_{i,1} + l_\xi \xi_{i,1}\right]\right)\right).$$

Now, we aggregate the above and use the fact that $\xi_{i,0}$ is a random walk and the fact that the household knows the local capital stock $k_{i,1}$ at period 0. We have:

$$\begin{aligned}\iota_0 = \frac{1}{\psi\delta}\{-R_0 &+ \left[(1-\beta(1-\delta))\frac{\phi}{\phi+1} + \left(\frac{(1-\beta(1-\delta))}{\phi+1}+\beta\right)l_\xi\right]y_0 \\ &+ \left[\left(\frac{(1-\beta(1-\delta))}{\phi+1}+\beta\right)l_k - (1-\beta(1-\delta))\frac{\phi}{\phi+1}\right]k_1\}\end{aligned} \tag{79}$$

We now turn to optimal utilization. First, note that, as the aggregate shock is i.i.d., the rational expectation of future utilization and future shadow value of capital are given by (78):

$$\mathbb{E}_0\left[u_1\right] = \Lambda_{uk}k_1 \quad \text{and} \quad \mathbb{E}_0\left[\vartheta_1\right] = \Lambda_{qk}k_1.$$

From the aggregate production (15), the optimal utilization (72), and the evolution of shadow value in (73), we have:

$$\vartheta_0 = -R_0 + \left(\beta\Lambda_{qk} - \tilde{\alpha}(1-\beta(1-\delta))(\Lambda_{uk}+1)\right)k_1. \tag{80}$$

Using (79), (80), the evolution of capital in (76), and the production (19), we can solve $(u_0, \iota_0, R_0, \vartheta_0)$ as a function of y_0 and k_0. In particular, we have

$$\iota_0 = \Gamma_{\iota y}y_0 + \Gamma_{\iota k}k_0,$$

where, again with the help of Mathematica, we can show that

$$\lim_{\nu\to+\infty,\phi\to 0}\Gamma_{\iota y} = \frac{\beta^3\delta^3 + \beta^2(3\delta-1)\delta + \beta(3\delta-1)+1}{\beta\delta\left(\beta^2\delta^2 + \beta\delta(\psi+2) - \beta\psi + \psi + 1\right)} > 0. \tag{81}$$

It follows that there exist $\bar{\phi}, \underline{\nu} > 0$ such that: whenever $\nu > \underline{\nu}$ and $\phi < \bar{\phi}$, investment and output comove when the informational friction is large enough.

To complete the argument, we must verify that consumption also comoves. Take the aggregate resource constraint (or goods market clearing):

$$y_0 = \frac{c^*}{y^*}c_0 + \left(1-\frac{c^*}{y^*}\right)(\iota_0 + k_0),$$

Solving this for c_0 and replacing our solution for investment, we have

$$c_0 = \frac{y^*}{c^*} y_0 - \frac{y^* - c^*}{c^*} \left(\iota_0 + k_0\right) = \left(\frac{y^*}{c^*} - \frac{y^* - c^*}{c^*} \Gamma_{\iota y}\right) y_0 - \frac{y^* - c^*}{c^*} \left(\Gamma_{\iota k} + 1\right) k_0.$$

From the above condition together with (81), we then conclude that, when ϕ, λ are small enough and ν and ψ are large enough (so $\Gamma_{\iota y} > 0$ and $\frac{y^*}{c^*} - \frac{y^* - c^*}{c^*} \Gamma_{\iota y} > 0$), output, investment, and consumption all positively comove in response to the aggregate demand shock. Finally, the comovement of employment is immediate, since

$$n_0 = \frac{\nu}{\nu + 1} y_0. \tag{82}$$

Two-period investment model.

Now, we provide a two-period version of the investment model in Section 6.4. This model can be solved analytically (without the help of Mathematica) and allows us to obtain a sharp necessary and sufficient condition for positive comovement between all key macroeconomic quantities (employment, output, consumption, and investment) in response to the aggregate discount rate shock.

In this two-period version of our model, at $t = 1$, households make investment, consumption, labor supply decisions,[4] and the firm makes utilization and labor demand decisions, as in Section 6.4. The capital still evolves according to (50). At $t = 2$, there are only consumption and labor decisions, but no investment and utilization. All capital depreciate completely after $t = 2$.

We log-linearize the equilibrium conditions and re-interpret all the variables as log-deviations from their counterparts in the deterministic equilibrium without aggregate and idiosyncratic shocks.[5]

The labor supply, labor demand, and the aggregate production in both periods (with $u_2 = 0$) are still given respectively by (71), (12), and (15). The optimal utilization at $t = 1$ is still given by (16). The shadow value of capital at the end of $t = 1$ is given by

$$\vartheta_{i,1} = -R_{i,1} + \mathbb{E}_t \left[y_{i,2} - k_{i,2}\right].$$

The optimal investment at $t = 1$ is still given by (74). The evolution of local capital is given by

$$k_{i,2} = -\frac{\delta'\left(u_1^*\right) u_1^*}{1 - \delta\left(u_1^*\right) + \iota_1^*} u_{i,1} + \frac{\iota_1^*}{1 - \delta\left(u_1^*\right) + \iota_1^*} \iota_1^i + k_{i,1}.$$

Aggregating the above conditions and using Proposition 5, the fact that $E_1^i\left[\xi_{i,2}\right] = \rho_\xi E_1^i\left[\xi_{i,1}\right] = \rho_\xi\left[y_{i,1} - E_1^i\left[y_1\right]\right]$, and that $k_1 = 0$, we have

$$\begin{aligned}
u_1 &= \frac{1}{\tilde{\alpha} + \phi} R_1 + \frac{\tilde{\alpha}}{\tilde{\alpha} + \phi} k_2 \\
\iota_1 &= \frac{1}{\psi \iota_1^*} \left(\left(1 - \lambda\right) \rho_\xi y_1 - \left[1 - \lambda\left(1 - \tilde{\alpha}\right)\right] k_2 - R_1\right) \\
k_2 &= -\frac{\delta'\left(u_1^*\right) u_1^*}{1 - \delta\left(u_1^*\right) + \iota_1^*} u_1 + \frac{\iota_1^*}{1 - \delta\left(u_1^*\right) + \iota_1^*} \iota_1.
\end{aligned}$$

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[4] The household's information at $t = 1$ is given by $\mathcal{I}_1^h = \left\{\beta_1^h\right\} \cup \left\{w_{h,1}, e_{h,1}, R_{h,1}, \left(p_{i,j,1}\right)_{i\in[0,1],j\in[0,1]}, k_{h,2}\right\}$.

[5] Here, in this deterministic equilibrium, we do not need to assume $k_1^* = k_2^*$. That is, we do not need to impose that $\delta\left(u_1^*\right) = \iota_1^*$. We only impose that, in this deterministic equilibrium, there is no adjustment cost. That is, $\Psi\left(\iota_1^*\right) = \iota_1^*$, $\Psi'\left(\iota_1^*\right) = 1$, and $\Psi''\left(\iota_1^*\right) = -\psi$.

Together, we have

$$\iota_1 = \frac{\left((1-\lambda) \left(\rho_\xi + \frac{\delta'(u_1^*)u_1^*}{1-\delta(u_1^*)+\iota_1^*} \right) - \frac{\tilde{\alpha}+\phi}{1-\tilde{\alpha}} \right)}{\psi \iota_1^* \left(1 + \frac{(1-\lambda)(1-\tilde{\alpha})}{\psi(1-\delta(u_1^*)+\iota_1^*)} \right)} y_1.$$

As a result, investment and output (and employment, similar to (82)) comove if and only if

$$(1-\lambda)\left(\rho_\xi + \frac{\delta'\left(u_1^*\right)u_1^*}{1-\delta\left(u_1^*\right)+\iota_1^*} \right) > \frac{\tilde{\alpha}+\phi}{1-\tilde{\alpha}}. \tag{83}$$

Using the aggregate goods market clearing, $y_1 = \frac{c_1^*}{y_1^*}c_1 + \frac{\iota_1^* k_1^*}{y_1^*}\iota_1$, we have that consumption and output comove if and only if

$$(1-\lambda)\left(\rho_\xi + \frac{\delta'\left(u_1^*\right)u_1^*}{1-\delta\left(u_1^*\right)+\iota_1^*} \right) - \frac{y_1^*}{k_1^*}\psi\left(1 + \frac{(1-\lambda)(1-\tilde{\alpha})}{\psi\left(1-\delta\left(u_1^*\right)+\iota_1^*\right)} \right) < \frac{\tilde{\alpha}+\phi}{1-\tilde{\alpha}}. \tag{84}$$

(83) and (84) together are necessary and sufficient for comovement among employment, output, investment, and consumption.

Finally, note that (83) is more easily satisfied when λ is small, underscoring the role of misperceptions in helping investment increase in response to η_t despite the increase in the real interest rate. On the other hand, (84) is more easily satisfied when ψ is large, underscoring how capital adjustment costs together with variable utilization allow aggregate employment and output to comove with aggregate consumption.

Appendix B: Monetary Extension

The starting point of our paper was the desire to accommodate the Keynesian narrative of demand-driven fluctuations outside the nexus of sticky prices and Phillips curves. But the mechanisms we have identified do not hinge on the absence of nominal rigidity: in its presence, they influence the properties of both the underlying natural rate of output and the output gap. We sketch the logic below.

Consider our baseline model and add sticky prices. Because our supply block abstracts from informational frictions, the standard derivation of the Phillips curve remains valid: inflation can still be expressed as a function of the output gap. What has changed, however, is the process for the natural rate of output, relative to which this gap must be calculated.

Turning to the demand side, Proposition 2 remains valid. But now the belief wedges $\mathcal{B}_t$ and $\mathcal{G}_t$ combine misperceptions about natural outcomes with misperceptions about output gaps, and they therefore depend on the conduct of monetary policy. To illustrate this point more, we next bypass any specific description of how monetary policy is conducted (e.g., a specific Taylor rule) and instead represent monetary policy in terms of "wedges."[6]

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[6] There is one subtlety here. To the extent that monetary policy does not stabilize the aggregate price level, its fluctuations may reveal the aggregate shock. To avoid perfect revelation, we can either assume that consumers are inattentive or introduce random consumption baskets along the lines of Lorenzoni (2009) and an earlier version of our paper. Namely, we could let each household be randomly matched to, consume the goods of, and observe the prices of, a non-representative sample of the islands in each period. This guarantees that the household would not learn the aggregate shock from observing

As well known (e.g., Correia, Nicolini, and Teles, 2008), introducing sticky prices is equivalent to maintaining flexible prices but allowing for a time-varying tax on labor and utilization, which is effectively under the control of monetary policy. Denoting this tax by τ_t, we have the following modification of Proposition 1 and 2.

Proposition 15 (AS and AD with sticky prices)**.** *Let τ_t denote a tax, or wedge, on labor and utilization. Aggregate supply is given by*

$$y_t = (1-\tilde{\alpha})(u_t + k_t) - \tau_t, \tag{85}$$

$$u_t = \frac{\beta}{\tilde{\alpha}+\beta\phi} R_t - \frac{\beta}{\tilde{\alpha}+\beta\phi}\left(\left(1+\frac{\nu+\sigma}{\tilde{\alpha}\sigma\nu}\right)\tau_t - \left(1+\beta\frac{\nu+\sigma}{\tilde{\alpha}\sigma\nu}\right)\mathbb{E}_t\left[\tau_{t+1}\right]\right) + \beta\mathbb{E}_t\left[u_{t+1}\right], \tag{86}$$

$$k_{t+1} = k_t - \kappa u_t. \tag{87}$$

Aggregate demand is given by

$$y_t = -\sigma\left(R_t + \beta_t\right) + \mathbb{E}_t\left[y_{t+1}\right] + \left(\mathcal{B}_t + \mathcal{G}_t\right), \tag{88}$$

where $\mathcal{B}_t$ and $\mathcal{G}_t$ are defined as (28) and (25).

Under this representation, a "hawkish" monetary policy that stabilizes inflation maps to $\tau_t = 0$, whereas an "accommodative" monetary policy that lets positive demand shocks trigger inflation and positive output gaps maps to a counter-cyclical τ_t: it is *as if* there is a subsidy on production whenever monetary policy is expansionary relative to the benchmark of replicating prices.

The textbook New Keynesian model, which abstracts from variable utilization, corresponds to either $\tilde{\alpha} = 1$ (utilization is unproductive) or $\phi \to \infty$ (variation in utilization is prohibitively costly). Aggregate supply then reduces to $y_t = 0 - \tau_t$, where 0 stands for the natural rate of output and τ_t for the wedge, or equivalently the output gap, induced by any monetary policy that does not replicate flexible prices. Relative to this familiar case, the key supply-side novelty of our analysis is to let the natural rate of output be sensitive to the real interest rate, in the manner explained in Section 4.

Let us now turn to aggregate demand, or equation (88) above. In the textbook version of the New Keynesian, this equation holds with $\mathcal{B}_t = \mathcal{G}_t = 0$. Relative to this case, we see that the informational friction continues to give rise to our two mechanisms, captured by the same terms $\mathcal{B}_t$ and $\mathcal{G}_t$ as in our baseline analysis. However, because the GE adjustment in the real interest rate is now modulated by monetary policy, the magnitude of $\mathcal{G}_t$ now depends on monetary policy.

Similarly, and more crucially for our narrative about confidence, $\mathcal{B}_t$ here contains not only misperceptions of the "natural" level of permanent income but also misperception of the output gaps induced by monetary policy. In particular, $\mathcal{B}_t$ can be decomposed as follows:

$$\mathcal{B}_t = \mathcal{B}_t^{natural} + \mathcal{B}_t^{gap},$$

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the prices of the islands it visits even when these prices comove with that shock. With the exception of the very last paragraph, in the remainder of this Appendix we ignore this subtlety and focus on how our mechanisms interact with monetary policy holding λ constant.

where

$$\begin{aligned}
\mathcal{B}_t^{natural} &\equiv \tfrac{1-\beta}{\beta} \sum_{k=0}^{+\infty} \beta^k \int E_t^h \left[\xi_{h,t+k}\right] dh \\
\mathcal{B}_t^{gap} &\equiv \bar{\mathbb{E}}_t \left[\mathcal{M}_t\right] - \mathcal{M}_t \\
\mathcal{M}_t &\equiv -\frac{1-\beta}{\beta} \sum_{k=0}^{+\infty} \beta^k \mathbb{E}_t[\tau_{t+k}]
\end{aligned}$$

$\mathcal{B}_t^{natural}$ is the value of $\mathcal{B}_t$ that obtains when monetary policy replicates flexible prices (equivalently, the value of $\mathcal{B}_t$ in our baseline analysis); $\mathcal{M}_t$ is a measure of how much monetary policy deviates from that benchmark; and $\mathcal{B}_t^{natural}$ is the corresponding average misperception, or equivalently the misperception of output gaps.

To put more structure on the new term, let us assume that monetary policy is such that

$$\tau_t = -\varphi\eta_t + \rho_\tau \tau_{t-1}, \tag{89}$$

where $\varphi \geq 0$ parameterizes the degree of policy accommodation, or the elasticity of the output gap with respect to the demand shock, and $\rho_\tau \in [0, 1)$ indexes its persistence. We then have that the gap between the actual present discounted value of the output gap and the average expectation of it is given by

$$\mathcal{B}_t^{gap} \equiv \bar{\mathbb{E}}_t \left[\mathcal{M}_t\right] - \mathcal{M}_t = -\frac{\varphi\,(1-\beta)}{\beta\,(1-\beta\rho_\tau)}(1-\lambda)\eta_t. \tag{90}$$

As long as $\lambda < 1$ and $\varphi > 0$, a positive aggregate demand shock therefore generates a negative value for $\mathcal{B}_t^{gap}$ at the same time that it generates a positive value for $\mathcal{B}_t^{natural}$.

What does this mean? As long as $\varphi > 0$, monetary policy lets output expand beyond its natural rate in response to a positive demand shock. This translates to an increase in *true* aggregate permanent income, via $\mathcal{M}_t$, which is perfectly forecasted under complete information $(\lambda = 1)$ but imperfectly so under incomplete information $(\lambda < 1)$. It follows that, as long as information is incomplete, consumers underestimate the increase in aggregate permanent income sustained by an accommodative monetary policy. And because the true increase in aggregate permanent income is larger when the output gaps induced by monetary policy are themselves larger (higher φ) or more persistent (higher ρ_τ), the size of belief mistake is larger under the same circumstances.

In this sense, an accommodative monetary policy goes *against* our confidence multiplier. But such a policy also *complements* our confidence multiplier by helping aggregate supply be more responsive to aggregate demand under sticky prices than under flexible prices. To see what we mean by this, let us shut down variable utilization. In this case, the flexible-price AS curve is vertical and the natural rate of output is invariant to aggregated demand shocks. It follows that, as long as monetary policy replicates flexible prices $(\varphi = 0)$, our confidence multiplier is switched off regardless of how large the informational friction is. But as soon as monetary policy is accommodative $(\varphi > 0)$, our confidence multiplier is active under sticky prices, even though it is inactive under flexible prices.

Perhaps more interestingly, our confidence multiplier helps amplify the power of monetary policy itself. To see this, abstract from the exogenous shock to consumer spending and, instead, modify (89) as follows:

$$\tau_t = -\eta_t^{MP} + \rho_\tau \tau_{t-1}, \tag{91}$$

where η_t^{MP} represents a pure policy shock, independent of any other shock in the economy. Then, while the informational friction dampens the effect of this shock via (90), it amplifies it via $\mathcal{B}_t^{natural}$.

We conclude with a comment on *optimal* monetary policy. In the textbook New Keynesian model, a monetary policy that stabilizes the price level is optimal becomes it minimizes relative price distortions (or other costs of inflation). But in our setting, a monetary policy that does the opposite could be desirable because it could let the variation in commodity prices reveal more information about the underlying state of the economy (and, thereby, increase λ and arrest our amplification). This suggests a novel policy trade-off, whose investigation we leave for future work.[7]

Proof of Proposition 15.

Since here we work directly with time-varying wedge (τ_t), we can still work with real prices and wages as in the main analysis, in the sense of denominated by the basket of all goods produced in the current period.

We define the production wedge as the as-if time varying tax on revenue reflected in the local firm's labor demand and utilization decisions:

$$\begin{aligned} l_{i,t} &= y_{i,t} - w_{i,t} - \tilde{\tau}_{i,t}, \\ p_{i,t} + q_{i,t} - u_{i,t} - k_{i,t} - \tilde{\tau}_{i,t} &= \vartheta_{i,t} + \phi u_{i,t}. \end{aligned} \tag{92}$$

The local labor supply in (13) is still given by:

$$n_t^i = \nu w_{i,t} - \frac{\nu}{\sigma} c_t^i.$$

Imposing labor market clearing, using the production function in (11), and aggregating, we have

$$y_t = (1 - \tilde{\alpha})(u_t + k_t) - \frac{\sigma\nu}{\sigma + \nu}\tilde{\alpha}\tilde{\tau}_t.$$

Defining $\tau_t = \frac{\sigma\nu}{\sigma+\nu}\tilde{\alpha}\tilde{\tau}_t$. We arrive at (85).

The shadow value of land is given by[8]

$$\vartheta_{i,t} = -R_{i,t} + (1-\beta)\,\mathbb{E}_t\left[p_{i,t+1} + q_{i,t+1} - k_{i,t+1} - u_{i,t+1}\right] + \beta\mathbb{E}_t\left[\vartheta_{i,t+1}\right]. \tag{93}$$

Aggregating (92) and (93) and combining terms, we have

$$y_t - (1+\phi)\,u_t - k_t - \tilde{\tau}_t = -R_t + (1-\beta)\,\mathbb{E}_t\left[y_{t+1} - k_{t+1} - u_{t+1}\right] + \beta\mathbb{E}_t\left[y_{t+1} - (1+\phi)\,u_{t+1} - k_{t+1} - \tilde{\tau}_{t+1}\right].$$

Similar to (20) but using the aggregate production in (85), we arrive at (86).

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[7] Note, though, that the presence of such a trade-off is likely to hinge on the informational-based interpretation of our setting. In the behavioral variants discussed in Section 6.5, there may or may not exist a relation between monetary policy and λ.

[8] Note that what matters for the shadow value of land is the actual marginal productivity of capital, which does not depend on the wedge $\tau_{i,t}$ directly.

The evolution of capital in (87) is the same as (21).

The derivation of the AD in (88) is exactly the same as the main analysis.

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